Segregation, Affirmative Action, and Investment

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Abstract

We study how individuals match in the workplace or in university when making side payments is costly (non-transferable utility or NTU). In contrast to the transferable utility (TU) benchmark, the pattern of matching will typically be inefficient, involving too much segregation, and provide a possible rationale for "associational redistribution" such as affirmative action: a social planner who could enforce a matching outcome that differs from the market outcome may raise aggregate social surplus. We show that this static inefficiency due to NTU can be exacerbated in a dynamic environment, in which investments made before the match determine individuals' productive types. In contrast to TU models, which always have an efficient equilibrium, there will typically be investment distortions, with high types over-investing and low types under-investing. We study several forms of associational redistribution, assessing the differential effects of achievement-based and background-based polices.

Keywords: Matching, nontransferable utility, affirmative action, segregation, education.

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1 Introduction

Some of the most important economic decisions we make – where to live, which profession to enter, whom to marry – depend for their consequences not only on our own characteristics or "types" (wealth, skill, or temperament), but also on those of the people with whom we live or work. These decisions matter not only statically, for our own well-being or those of our partners, but also dynamically: the prospect of being able to select particular kinds of neighbors, associates or mates, or the environment those partners provide, affect the costs and benefits of investment. The impact of those investments may extend far beyond our immediate partners to the economy as a whole.

A natural question – one in which policy makers in rich and poor countries have taken a direct interest – is whether the market outcome of our "matching" decisions leads to outcomes that are socially desirable. In fact, it has been often contended in public policy debates in the U.S., U.K., India and elsewhere that the market often *has* failed to sort people desirably: there is too much segregation (by educational attainment, ethnic background or caste); certain groups appear to be "excluded" from normal participation in economic life; and that in turn depresses their willingness to invest in human capital. If the market does mismatch people in this way, policy remedies might include "re-matching" individuals into other partnerships via affirmative action or school integration.

Economic theory makes it clear that some form of imperfection needs to be present in order to rationalize such policy intervention.[1] In this paper we will explore the ramifications of one such imperfection – non-transferable utility (NTU) within matches. For a number of plausible reasons this assumption may often be pertinent. Imperfections in the credit market, or moral hazard stemming from limited observability of effort are among the more important one. Others are incomplete contracts, commitment problems leading to renegotiation, limited observability of firm profits, risk aversion, legal constraints

[1] If the characteristics of matched partners (ability, gender, or race) are exogenous, then under the assumptions that (1) partners can make non-distortionary side payments to each other (transferable utility or TU); (2) there is symmetric information about characteristics; and (3) there are no widespread externalities, stable matching outcomes are social surplus maximizing: no other assignment of individuals can raise the economy's aggregate payoff. Even if characteristics (such as income or skill) are endogenous, the result of investments made either before matching or within matches, under the above assumptions re-matching the market ooutcome is unlikely to be desirable (Cole et al., 2001, Felli and Roberts, 2002).

and regulation, or "behavioral" reasons such as envy or inequity aversion. These problems are likely to arise not only in professional firms with profit sharing arrangements, where effort observability and commitment are likely to be problematic, but also in industrial firms offering some form of efficiency wages. Moreover, when part of compensation is inalienable, such as training or reputation, transferring individual gains may become very costly. Despite the importance of NTU in many parts of economics, the its implications for the nature of market matches, the level and distribution of investment in such markets, and the effects of re-matching policy have not been given much attention.

This is in stark contrast to other imperfections, such as search frictions, widespread externalities, and statistical discrimination, which have all been studied as possible sources of matching market failure that can generate inefficient levels of output and investment, or undesirable degrees of inequality. The latter in particular has been cited as a justification for policy intervention that directly interferes with the sorting outcome through re-matching, that is *associational redistribution* (AR) (Durlauf, 1996a). Examples include affirmative action (see Fryer and Loury, 2007), school integration, or certain types of labor subsidies that target the less qualified. AR has also been supported on efficiency grounds, in the case where there is a problem of statistical discrimination: Coate and Loury (1993) provides one formalization of the argument that equilibria where under-investment is supported by "wrong" expectations may be eliminated by affirmative action policies (an "encouragement effect"), but importantly also points out a possible downside ("stigma effect").

In addition to the gap in theoretical understanding, there some empirical reasons why the NTU case also deserves study. Most striking is that the removal of AR policies that have been in place for a while often results in reversion to the pre-policy status quo. This seems hard to reconcile with incorrect beliefs, as well with as search frictions, as the fundamental source of mis-match. NTU also provides a natural explanantion for political opposition to AR: it is difficult to compensate the unfavored group; if not the market would have already done so.

It is known at least since Becker (1973) that under nontransferable utility the equilibrium matching pattern need not maximize aggregate social surplus (see also Legros and Newman, 2007). This is because the types that may have received large shares of the pie generated in an (efficient) match under

TU, will now receive a smaller share to due to rigidities in dividing that pie, and so they will prefer to match with types with whom they can obtain large payoffs. In some cases this will lead to what appears as excessive segregation.

It is useful to distinguish among three distinct but interacting distortions that occur in NTU matching models. We refer to them as inefficiency *of* the match, *by* the match, and *for* the match. *By-the-match* inefficiency results when the Pareto frontier for matched agents does not coincide with an iso-surplus surface; matched partners need not maximize their own joint surplus, and aggregate performance is sensitive to the distribution of surplus within matches (see e.g., Legros and Newman, 2008). As we have suggested, it may cause *Of-the-match* inefficiency, which refers to the kind of mis-match pointed out by Becker, wherein reassignments of partners may raise the aggregate welfare.

For-the-match inefficiency results from the first two: since surplus shares and levels are distorted in a laissez-faire match, so are incentives to invest before it happens. In particular, NTU inhibits the market's ability to attract an efficient distribution of skills by signaling scarcity through adjustments in wages or other surplus shares. Returns to skills in short supply may not rise enough to encourage investment, while returns to plentiful skills may remain too high, encouraging overinvestment.

Thus NTU provides an efficiency-based rationale for AR policy, at least if one accepts an "ex-ante" Pareto optimality criterion, i.e., maximizing welfare from behind a veil of ignorance, before people know their types (as in Harsanyi, 1953, Holmström and Myerson, 1983).[2] Though it cannot directly address the sources of NTU and therefore by-the-match inefficiency, it does provide an instrument for correcting inefficiency of the match. The mis-match that associational redistribution could help to correct here does not depend on asymmetry of information about types or a concomitant role of (self-fulfilling) beliefs about the productivity of individuals with observable attributes that may be correlated with type. A possible pitfall of such policies – one that has often been made in policy debates – is that they may harm the investment incentives of the group favored by the policy by

[2] Associational redistribution typically does not yield Pareto improvements unless accompanied by compensation, that is monetary transfers, which are, by nature, severely limited in a nontransferable utility framework. Nevertheless we will evaluate allocations in terms of aggregate surplus. This can be defended on the grounds of taking an ex-ante perspective before agents' types have realized, a view that e.g., potential parents may take when voting on educational policy.

guaranteeing its members minimal payoffs. Moreover, they may reduce the incentives of *unfavored* groups, whose members may get high returns under laissez faire. Against this is the possibility that a properly designed AR policy results in payoffs that more closely replicate the TU outcome, so that efficient investment incentives might be partially restored. In any case, an overall assessment of AR policy must therefore take account of their effects on investment incentives, i.e., on whether they mollify or exacerbate for-the-match inefficiency.[3]

In general, the interaction of all three distortions must be taken into account for assessing policy; here, we shall shut down inefficiency by the match in order to focus on the other two distortions by assuming *strict* NTU, i.e., the Pareto frontier is a single point. This also allows us to focus on redistributive policies that are purely associational, since transfers such as taxes and subsidies would be hard to implement in this environment.[4]

The setup we employ to analyze various forms of associational redistribution is as follows. Agents have a binary background type reflecting whether they are *privileged* or not. Privilege confers a productivity benefit, either in terms of (increased) labor market output or (reduced) education cost. Agents can affect their labor-market productivity (also a binary variable) by investing in education, which determines the probability of becoming a high-achievement type. In the labor market agents match into firms whose output depends on members' achievement and possibly background. The production technology is such that diversity (heterogeneity) within firms is more productive, and would be the outcome under unrestricted side payments. We model nontransferable utility in the simplest possible way: output is shared equally within firms.

As a result the labor market segregates in educational achievement and background. Thus, the laissez faire equilibrium outcome is inefficient from an aggregate surplus perspective. When agents' types enter the production function directly, returns from education investment depend positively on the quality of the match an agent obtains on the labor market. Then there is likely to be simultaneous *overinvestment at the top* and *underinvestment*

[3]And quite apart from whether such policy is desirable, it is of interest to predict its likely effects; for instance, variations in aggregate surplus in the model may correspond to variations in national income across countries following different policies.

[4]However, they may be approximated to varying degrees by allowing for randomized matching policies; here we will focus on deterministic policies and defer consideration of the broader menu of policies to another paper.

at the bottom: the underprivileged find investing to be too costly or unremunerative, while the privileged receive inefficiently high rewards in the labor market.

We then evaluate several associational redistribution policies. When labor market segregation in education is inefficient, an immediate remedy is an *achievement based* policy rematching agents based on educational attainment. For instance, "Workfare" and certain European labor market policies provide wage subsidies for employing long-term unemployed or low educated youth. But in a dynamic setting with investments, a trade-off emerges. Though rematching increases output, investment incentives are depressed: the policy raises the returns to low education outcomes and lowers the return to high outcomes. The adverse incentive effect may be partially mitigated by a rematching policy that conditions not on results of choices but on exogenous information correlated with education outcomes, such as agents' socioeconomic status. Examples of such *background based* policy are race- or gender-based affirmative action.

The literature on school and neighborhood choice (see among others Bénabou, 1993, 1996, Epple and Romano, 1998) typically finds too much segregation in types. This may be due to market power (see e.g., Board, 2008) or widespread externalities (see also Durlauf, 1996b, Fernández and Rogerson, 2001). When attributes are fixed, aggregate surplus may be raised by an adequate policy of bribing some individuals to migrate (see also de Bartolome, 1990). Fernández and Galí (1999) compare matching market allocations of school choice with those generated by tournaments: the latter may dominate in terms of aggregate surplus when capital market imperfections lead to nontransferabilities. They do not consider investments before the match.

Peters and Siow (2002) and Booth and Coles (2009) present models where agents invest in attributes before matching on a marriage market under strictly nontransferable utility. Investments are taken in solitude, so peer effects are absent. The former finds that allocations are constrained Pareto optimal (with the production technology they study, aggregate surplus is also maximized), and does not discuss policy. The latter compares different marriage institutions in terms of their impact on matching and investments. Gall et al. (2006) analyze the impact of timing of investment on allocative efficiency. Several recent studies consider investments before matching under asymmetric information (see e.g., Bidner, 2008, Hopkins, 2005, Hoppe et al., 2009), mainly focusing on wasteful signaling, while not considering associational redistribution.

Finally, the emphasis here is on characterizing stable matches and contrasting with ones imposed by policy. Thus we shall not be concerned here with the market outcome under search frictions (Shimer and Smith, 2000, Smith, 2006), nor on mechanisms that might be employed to achieve either stable matches or ones with desirable welfare properties (e.g., Roth and Sotomayor, 1990). Stable matches in this paper may, of course, be attained using matching mechanisms.

The paper proceeds as follows. Section 2 provides a first look at the effectiveness of AR policies in terms of sorting, incentives, and exclusion in a world where agents are homogenous. Section 3 considers the general case where agents can be distinguished by background and where AR policies can be based on achievement or on background. Section 4 concludes, and the appendix contains the more tedious calculations.

2 A First Look at Associational Redistribution

The market is populated by a continuum of identical agents I with unit measure. Though we refer to it as a "labor market," it can also be interpreted in other ways, for instance as a market for places in university. Agents are characterized by their educational attainment a, which is either high h or low ℓ (in the university interpretation, these would be secondary school achievements). Denote the measure of h agents by $q \in [0, 1]$. In the market, agents match into firms of size two and jointly produce output. Profit $y(s, s')$ in a firm depends on agents' skill levels s and s'. An agent's skill is for now given by the education outcome, $s = a$. Assume that profits increase in skill,

$$y(\ell, \ell) < y(\ell, h) = y(h, \ell) < y(h, h).$$

Profits in firms have the *desirability of diversity* property if

$$2y(h, \ell) > y(h, h) + y(\ell, \ell). \tag{DD}$$

DD holds for instance when a is real-valued and y a concave function of the sum of the types. It could also be the result of a technology that combines two different tasks, one human capital intensive and one less so, say engineering and design versus actual manufacturing, with the firm free to assign the worker to the task (Kremer and Maskin, 1996, Legros and Newman, 2002).

To provide a benchmark solve now for the competitive labor market equilibrium, that is a stable match of agents into firms. With DD there are wages $w(h,\ell) \geq 0$ and $w(\ell,h) \geq 0$ with $w(h,\ell) + w(h,\ell) = y(h,\ell)$ such that

$$w(h,\ell) \geq y(h,h)/2 \text{ and } w(\ell,h) > w(\ell,\ell)/2.$$

This implies that given wages $w(.)$ there is no distribution of profits in segregated firms such that agents in integrated firms were better off forming a segregated firm. Hence, in labor market equilibrium measure $\min\{q, 1-q\}$ of integrated firms emerge, the remainder segregates. Market wages are determined by scarcity, that is $w(h,\ell) = y(h,h)/2$ if $q > 1/2$, $w(\ell,h) = y(\ell,\ell)/2$ if $q < 1/2$, and $w(h,\ell) \in [y(h,h)/2, y(h,\ell) - y(\ell,\ell)/2]$ if $q = 1/2$.

2.1 Nontransferable Utility

The example above tacitly assumed that utility was perfectly transferable on the labor market. This means agents can contract on the distribution of profit in a firm without affecting productive efficiency, i.e., firm profit. For a number of plausible reasons this assumption may often be violated. Lack of access to or imperfections on the credit market, limited liability and moral hazard within the firm are one reason not to expect perfectly transferable utility. Others are incomplete contracts and renegotiation, risk aversion, legal constraints and regulation, or behavioral concerns.

To facilitate exposition we assume an extreme case of non-transferabilities, namely strictly nontransferable utility, so that only a single vector of payoffs to agents is feasible in any firm.[5] To minimize notation, assume that profits are shared equally in firms, and abbreviate

$$y(h,h) = 2W, y(h,\ell) = y(\ell,h) = 2w, y(\ell,\ell) = 0.$$

Assume as a normalization that $W < 1$. Then wages are typically bounded above by 1, which permits interpretation of investments induced by expected wages, introduced below, as probabilities.

Nontransferable utility affects the equilibrium labor market allocation quite substantially. Despite $2w > W$, i.e., DD holds, integration is no longer possible in equilibrium. Suppose that a positive measure of (h,ℓ) firms form

[5] All results in the paper are robust to allowing for some transferability by admitting for either a sufficiently small amount of perfect transferability, or for sufficient curvature in the Pareto frontier within teams, see Appendix.

and h agents obtain wage w. Then any two h agents have a profitable deviation by starting a (h, h) firm earning W each, a contradiction to stability. Hence, under strictly nontransferable utility only homogeneous firms emerge.

If both high and low types have positive measure ($0 < q < 1$), and diversity is desirable in production ($2w > W$), aggregate surplus is strictly lower when utility is nontransferable. If $q \leq 1/2$, surplus is $2qw$ if utility is transferable, while it is qW if not; if $q > 1/2$, surplus is $(1-q)2w+(2q-1)W$ if utility is transferable, and qW if not.

Non-transferability of utility may therefore distort the matching pattern and reduce aggregate surplus. This seems to provide a first-order rationale for associational redistribution on the labor market. Moreover, studying such interventions may yield insights about the output effects of policies.

Consider a policy of associational redistribution (AR) that assigns h agents to ℓ agents whenever possible. Any remaining agents are rationed uniformly into homogeneous firms. This policy replicates the matching pattern under transferable utility and achieves an increase in aggregate surplus for any exogenously given distribution of educational attainments, as measured by q in our example. However, the non-favored group of type h is clearly worse off under the policy.

Active labor market policy often resembles such policy, e.g., employment subsidies or mentoring programs. By targeting the long term or young unemployed such policy effectively rematches the labor market conditional on educational achievements or rather lack thereof. In many industrialized countries some form of wage subsidy or workfare program was used: the Targeted Jobs Tax Credit (TJTC) and later on the Work Opportunity Tax Credit (WOTC) in the US, as part of the Hartz policy reform in Germany, in the New Deal for Young People in the UK, and payroll tax subsidies for minimum wage labor contracts and wage subsidies for unemployed youth in France.

2.2 Education Investments

An often voiced critique of associational redistribution is that it spoils incentives. Educational attainments, and agents' attributes on markets in general, often result from individual choices, which are affected by the anticipated rewards accruing to the various types. Therefore endogenizing types is a natural way to assess such critique.

When investing, agents exert effort $e \in [0, 1]$ to acquire education. Effort

e, which is never verifiable, comes at a utility cost $e^2/2$ and determines the probability of high skill as the education outcome. When spending effort e, an agent attains high skill h with probability e and a low skill ℓ otherwise. The measure of high achievers (agents with attainment h), q, is now endogenous and given by $q = e$.

Events unfold as follows.

- Agents simultaneously choose e.
- Educational outcomes are realized and publicly observed.
- Agents form a stable match (in case of laissez faire) or are matched in accordance with the AR policy.

Let $w(h)$ and $w(\ell)$ be the rationally anticipated wages of a high and a low achiever. Then, an agent solves

$$\max_e e[w(h) - w(\ell)] + w(\ell) - \frac{1}{2}e^2,$$

yielding $e = w(h) - w(\ell)$.

In the benchmark case, when utility is fully transferable, investment incentives depend on whether q is anticipated to be greater or lower than $1/2$. If $q > 1/2$, high achievers are in excess supply and have a chance of being matched with a low or a high achiever. As a pair of high achievers obtain $w(h) = W$ each, this is also what they get when matched with a low achiever,[6] who gets the residual $w(\ell) = 2w - W$. Hence, investments are $e = 2(W - w)$. If $q < 1/2$, low achievers are in excess supply, obtain $w(\ell) = 0$, and high achiever match only with low achievers obtaining $w(h) = 2w$. Investments are $e = \min\{2w; 1\}$, strictly greater than in the case $q > 1/2$. In equilibrium the anticipated q coincides with its realization e, for instance, if $q > 1/2$, $e^T = 2(W - w)$ and therefore we need that $W - w > 1/4$. We have the following result (all proofs missing in the text are in the appendix).

Lemma 1 *Suppose that utility is fully transferable.*

(i) If $W - w > \frac{1}{4}$, $e^T = 2(W - w)$ *and* $q > 1/2$,

(ii) If $W - w < \frac{1}{4} < \min\{w; 1/2\}$, $e^T = \frac{1}{2}$ *and* $q = 1/2$,

[6] Equal treatment on the labor market holds under transferability: if one high achiever gets strictly more than another one, the latter can match with the partner of the former at a wage that is slightly lower.

(iii) If $\min\{w; 1/2\} < \frac{1}{4}$, $e^T = \min\{2w; 1\}$ *and* $q < 1/2$.

When utility is strictly nontransferable, agents segregate on the labor market by achievement level. Therefore $w(h) = W$, $w(\ell) = 0$, and the laissez-faire investment is $e^L = W$. By Lemma 1 the social return from education, and thus e^T, decreases with q. Under nontransferable utility the private return from education is independent of q. Hence, given W, the difference in investment under nontransferable and transferable utility increases in q, and therefore in W.

Corollary 1 *Comparing investment levels when utility is perfectly transferable (TU) and strictly nontransferable (LF) yields*

$$e^L > e^T \Leftrightarrow W > \frac{1}{2}.$$

Thus, both *overinvestment* relative to the benchmark (if $W > 1/2$), since $W > 2(W - w)$ and *underinvestment* (if $W < 1/2$) are possible.

2.3 Associational Redistribution

Since mismatches due to nontransferable utility distort investment incentives, the case for associational redistribution seems even more compelling when the measure of high achievers is endogenous. This intuition is incomplete, however, because *given nontransferabilities on the labor market* enforcing the "correct" sorting may in fact worsen investment incentives substantially.

Recall that under transferable utility the labor market wage adjusts as to provide the long market side with its autarky payoff (i.e., W for h and 0 for ℓ agents). For instance, if low achievers are in excess supply they obtain 0 and high achievers get $2w$. If utility is strictly nontransferable, however, low achievers have strictly positive payoff under an achievement based policy, since they get w with a positive probability due to uniform rationing. High achievers get w for sure. Hence, investment incentives are weaker than under laissez-faire as the return is lower in the high achievement state h and higher in the low achievement state ℓ. Indeed, it is not hard to show that in any equilibrium under forced integration low achievers must be in excess supply.

Proposition 1 *Under an AR policy the measure of educated agents is less than* $1/2$. *Investment in education, and the proportion of high achievers, is*

$$e^A = w - \frac{1}{2}\left[\sqrt{4w^2 + 1} - 1\right]$$

Clearly, $e^A < w < W = e^L$ and forcing integration on the labor market worsens investment incentives. But since firms produce more output under integration than under segregation, rematching has also a positive effect on aggregate surplus. Whether an AR policy improves upon the laissez-faire allocation thus depends on whether the gain in output is large enough to offset investment distortions. Aggregate surplus under laissez-faire is $S^L = W^2/2$. An AR policy induces a total surplus of

$$S^A = e^A \left(2w - \frac{e^A}{2}\right).$$

Therefore the achievement based policy improves on laissez-faire when

$$\underbrace{e^L(2w - W)}_{\text{output gain given l-f effort}} > \underbrace{(e^L - e^A)2w}_{\text{output loss given rematch}} \underbrace{- \frac{1}{2}((e^L)^2 - (e^A)^2)}_{\text{savings in costs}}. \tag{1}$$

The LHS captures the surplus added by integration under the AR policy keeping investment at its laissez-faire level. The RHS measures the effects on investment: a lower output given the rematch and a savings in cost due to lower incentives. Straightforward calculation shows there exists a unique value of W for which condition (1) holds with equality.

Corollary 2 *There is an increasing function $W_0(w)$, such that total surplus under an AR policy is higher than under laissez-faire if and only if $W \leq W_0(w)$.*

The cutoff $W_0(w)$ is less than $2w$ (as W approaches $2w$ diversity gains vanish, and incentive losses outweigh them) and increases in w (for larger values of w investment incentives weaken under AR).

The analysis of this simple model illustrates how non transferabilities prevent the market from allocating skills efficiently, both in from static and dynamic perspectives. Statically, NTU causes mismatches because types cannot be paid at their opportunity cost (segregation payoff) in integrated firms. Dynamically, since payoffs are distorted so is investment as the labor market does not allow the proper signaling of scarcity. Therefore agents may over- or under-invest with respect to the TU benchmark.

In this framework, AR policies improve static efficiency *for a given distribution of skills*, since they force integration and therefore reduce the number of mismatches in the labor market. However, because the sharing of the surplus is still subject to non-transferabilities, AR policies may also worsen dynamic efficiency, rendering the comparison between AR and laissez-faire policies non trivial.

These effects are present when agents are homogeneous, but often markets segregate not on endogenous choices such as school achievement, but on exogenous characteristics like gender or race. Policies usually aim to remedy both excessive segregation on the basis of achievement and of other characteristics. Ramsey taxation logic suggests that AR policies based on these (exogenous) characteristics like gender or race might work better than those based on (endogenous) variables like school achievement. Nevertheless, as we will see in the analysis of the general model, the dynamic effects of these policies on investment incentives may overturn this simple intuition.

3 AR Policies with Heterogenous Agents

3.1 Heterogeneity in Background

We consider now the general case where agents differ in their exogenous characteristics, which may affect behavior differentially at the investment stage and the labor market stages. Participation in education is typically subject to individual choice and labor market returns to education govern the extent of exclusion, since low returns discourage agents from investing. Returns are likely to be affected by individual socio-economic characteristics of agents' neighborhoods or parents, race, or gender, for instance through the cost of accessing education or the profit attained in a firm. These characteristics can be thought of as an agent's type, or *background* b. Suppose that $b \in \{u, p\}$, indicating whether an agent has a *privileged* or an *underprivileged* background. The measure of agents with background p is exogenous and denoted by π; the remainder $1 - \pi$ agents have background u.

Each agent can exert effort e in order to become a high achiever with probability e. In principal an agent's background could interfere with access to education opportunities, or affect payoffs resulting from investment and the subsequent activity in the labor market. At the time the labor market opens, an individual's skill is now a function of both achievement and background and "types" in the labor market are two-dimensional. An agent with background $b \in \{u, p\}$ can therefore have types (ℓ, b) and (h, b).

Returns from joint production are a function of the "skills" of each partner, where the skill is a function of achievement and background. The set of skills is then

$$S = \{\ell u, hu, \ell p, hp\}.$$

Output in a firm is determined by the function $y(s, s')$ with $s, s' \in S$.

This formulation is consistent with agents of different backgrounds facing the same investment technology. That is, the distribution of achievement levels (ℓ, h) as a function of effort is the same for both backgrounds. Background does affect the productivity of agents: this can be due to differences in culture, access to social networks, knowledge about markets and the like. In this interpretation, 'skills' are two-dimensional; we will assume that there is a complete order on S.

Note that skill could also be one-dimensional, for instance when background affects the probability of obtaining a particular achievement level for a given level of effort.

In either interpretation, a special case is when $\ell u = hu = \ell p < hp$: underprivileged agents have no incentive (in the two-dimensional case) or possibility (in the one-dimensional case) to invest, since their productive skill s is independent of their achievement. Then only a proportion π of agents (the privileged) may profitably invest to become high achievers, while a proportion $1 - \pi$ of agents (the underprivileged) have no profitable investment opportunity, become low achievers and therefore "excluded". This case is a simple extension of the model of section 2, with the caveat that in addition to an AR policy based on achievement only, it is possible to think of an AR policy based on background. As the underprivileged have no profitable investment opportunity, investment distortions can only arise among the privileged. When the underprivileged have a profitable investment opportunity, the effect of NTU on investment incentives may differ between types. This leads to one of the main results of this section in that the privileged tend to over-invest while underprivileged tend to under-invest.

Our results below require $\ell u \leq \ell p$ and $hu < hp$. That is, both underprivileged low and high achievers have lower skills than their privileged counterparts.To facilitate exposition we will focus on the case where all underprivileged's skills are less than that of the lowest skilled privileged agent:

$$\ell u < hu \leq \ell p < hp.$$

We assume throughout that $y(.)$ is monotone in its arguments ($y(s, s') > y(s, s'')$ if $s' > s''$).[7]

[7] A weaker form of monotonicity, $y(s, s') < \max\{y(s, s); y(s', s')\} \leq y(hp, hp)$ for all

Nontransferabilities cause segregation, for the same reason as in Section 2: agents maximize their payoff given the match and therefore always favor high skilled partners. Clearly, under laissez-faire investment distortions arise only if segregation is not the benchmark outcome under full transferability. For this reason we impose that diversity is indeed desirable:

$$2y(s, s') > y(s, s) + y(s', s') \tag{DD}$$

for all $s \neq s' \in \{\ell u, hu, \ell p, hp\}$. To be able to interpret investments as probabilities let the maximum difference in segregation payoffs is bounded above by 1, i.e.

$$y(hp, hp) - y(\ell u, \ell u) < 2.$$

The next two assumptions reduce the number of cases we need to consider in our analysis. Both put bounds on the returns to education for different backgrounds and the marginal benefit of one variable when there is already integration in the other variable.[8]

First, we assume that education has a discouragingly low return in a firm made up of only underprivileged agents:

$$y(hu, hu) - y(\ell u, \ell u) < \min\{1; y(\ell u, hp) + y(hu, \ell p) - 2y(\ell u, \ell p))\}. \tag{DC}$$

The next condition requires that integrating in one dimension (here background) exhausts the gains from diversity,[9]

$$\begin{aligned} &y(hu, hp) + y(\ell u, \ell p) > y(\ell u, hp) + y(hu, \ell p) \text{ and} \\ &y(hu, hp) + y(\ell u, \ell p) > y(\ell p, hp) + y(\ell u, hu), \end{aligned} \tag{M}$$

This last assumption will favor AR policies based on background since the usual cost of these policies – to allow segregation in achievement – is not longer present. Despite this advantage, achievement based policies will continue to dominate for some distributions of backgrounds in the population.

This class of production environments is, for instance, consistent with the task assignment technology in Kremer and Maskin (1996). Another application that has this property are problem-solving tasks where groups with diverse backgrounds tend to perform well due to heterogeneous heuristics

$s \neq s'$ is sufficient.

[8]The weak versions of conditions (DC) and (M) hold trivially if $\ell u = hu = \ell p < hp$.

[9]When instead full integration is desirable results are very similar to the extension of the model of section 2 when there is a proportion of agents who cannot invest.

(Hong and Page, 2001). The results carry over to cases when mixing heterogeneous types in firms is subject to frictions, for instance due to different languages. Property (DD) postulates that surplus gains outweigh productive or utility losses from diversity. Likewise, when transferability of utility is higher among agents of similar backgrounds our results continue to hold, since in this case heterogeneity in achievement can be supported when the market segregates in background, which by Property (M) is surplus-inferior to integration in backgrounds and segregation in achievements (e.g. when the beauty is not quite a beast in Legros and Newman, 2007).[10]

3.2 Laissez-faire versus Full TU Benchmark

Laissez-faire (NTU)

The laissez-faire equilibrium allocation under strictly nontransferable utility has full segregation in skill types, as above. To see this suppose there is a positive measure of (s, s') firms, with $s \neq s'$. This cannot be part of an equilibrium since $\max\{y(s,s), y(s',s')\} > y(s,s')$ by monotonicity of the function $y(.)$. Hence, in equilibrium under laissez-faire the economy segregates in skill types yielding payoffs

$$w(s) = \frac{1}{2}y(s,s).$$

This implies investments

$$e_p^L = \frac{1}{2}(y(hp,hp) - y(\ell p,\ell p)) \text{ and } e_u^L = \frac{1}{2}(y(hu,hu) - y(\ell u,\ell u)).$$

Full TU Benchmark

When utility is fully transferable measures of types ℓu, hu, ℓp, and hp in the labor market are $(1-\pi)(1-e_u)$, $(1-\pi)e_u$, $\pi(1-e_p)$, and πe_p, where e_u^T and e_p^T denote (endogenous) investment level of u and p agents. Investment is determined by market wages for skill types $w(s)$:

$$e_u^T = w(hu) - w(\ell u) \text{ and } e_p^T = w(hp) - w(\ell p).$$

Since $w(s) + w(s') = y(s,s')$, equilibrium wages are pinned down by the types of firms (s,s') that have positive measure in equilibrium. For instance,

[10] y can also be interpreted as a social welfare function (especially if the labor market is university). Then property (DD) may reflect a desire for egalitarianism or to make up for past wrongs.

suppose that skill type hp has measure greater than 1/2, i.e., $\pi e_p > 1/2$. Property (DD) implies that an equilibrium allocation has to exhaust all potential opportunities for integration, and there can be at most one skill type s such that (s, s) firms have positive measure. Hence, an equilibrium allocation under full TU must have positive measure of matches (hp, s') for each $s' = \ell u, hu, \ell p, hp$ with positive measure.

Because hp agents are in excess supply, they will obtain their segregation payoff in equilibrium, that is $w(hp) = y(hp, hp)/2$. Therefore ℓp agents obtain $w(\ell, p) = y(hp, \ell, p) - y(hp, hp)/2$ and the wage premium for a privileged agent who becomes high achiever is $w(hp) - w(\ell p) = y(hp, hp) - y(hp, \ell, p)$.

This gives equilibrium wages and investments as

$$e_u^T = y(hu, hp) - y(\ell u, hp) \text{ and } e_p^T = y(hp, hp) - y(\ell p, hp).$$

This is consistent with $\pi e_p > 1/2$ if and only if $\pi(y(hp, hp) - y(\ell p, hp)) > 1/2$. Property (M) implies that the skill type an agent is matched to weakly increases in that agent's own type. This determines the equilibrium allocation whenever no skill type has measure greater than 1/2. Table 1 gives a summary of the possible equilibria and lists market wages and corresponding investment choices e_u^T and e_p^T associated to the different regimes. Market wages are obtained by solving the system of equations given by $w(s) + w(s') = y(s, s')$ for all firms (s, s') with positive measure.

Overinvestment at the Top and Underinvestment at the Bottom

When background affects production an interesting case may emerge.

Returns to education (given by $w(hu) - w(\ell u)$ and $w(hu) - w(\ell u)$) under nontransferable utility may exceed the benchmark for the privileged and fall short of it for the underprivileged, implying overinvestment at the top and underinvestment at the bottom. This is because under desirability of diversity (DD)

$$y(\ell u, hu) - y(\ell u, \ell u) > \frac{1}{2}(y(hu, hu) - y_(\ell u, \ell u)) > y(hu, hu) - y(\ell u, hu),$$

and

$$y(\ell p, hp) - y(\ell p, \ell p) > \frac{1}{2}(y(hp, hp) - y_(\ell p, \ell p)) > y(hp, hp) - y(\ell p, hp),$$

If privileged high achievers hp are relatively abundant on the labor market while underprivileged high achievers hu are scarce, benchmark returns to education are $w(hu) - w(\ell u) = y(\ell u, hu) - y(\ell u, \ell u) > e_u^L$ and $w(hp) - w(\ell p) = y(hp, hp) - y(\ell p, hp) < e_p^L$. Indeed payoffs and investments in Table 1 imply the following.

Proposition 2 *(i) u agents never overinvest in education.*

(ii) There is $\underline{\pi}$ such that for all $\underline{\pi} < \pi \leq 1$ under laissez-faire privileged agents overinvest ($e_p^L > e_p^T$), and underprivileged agents underinvest ($e_u^L < e_u^T$). The threshold $\underline{\pi}$ is given by $\underline{\pi} = 1/(y(hp, hp) - y(\ell p, \ell p))$.

For (i) consider the case $\pi < 1/2$ and assume by way of contradiction that u agents with achievement h have measure greater than 1/2. Investment in education for u agents under full TU is

$$e_u^T = y(hu, hu) - y(\ell u, hu) < \frac{1}{2}(y(hu, hu) - y(\ell u, \ell u)) = e_u^L.$$

However, condition (DC) requires $(1 - \pi)(y(hu, hu) - y(\ell u, \ell u)) < (1 - \pi)$ so that $(1-\pi)e_u^T < 1/2$ and implies wages equal to $w(hu) - w(\ell u) \neq y(hu, hu) - y(\ell u, \ell u)$, a contradiction. Under (DC) $y(hu, hu) - y(\ell u, \ell u) < 2(y(hu, s) - y(\ell u, s))$, for $s \neq hp$ which together with (M) implies that the underprivileged never overinvest. The case $\pi > 1/2$ leads to the same conclusion.

3.3 Achievement versus Background Based AR Policies

Achievement Based AR

Suppose an achievement based policy is introduced into this setting. That is, under the policy all potential matches between h and ℓ agents must be exhausted, but given this constraint agents are free to match in backgrounds as they prefer. Note first that under an achievement based policy high achievers remain scarce.

Proposition 3 *Under an achievement based policy less than half of the privileged and the underprivileged become educated, $e_u^A < 1/2$ and $e_p^A < 1/2$. Investment is given by*

$$e_b^A = \frac{1}{2} + \frac{\Delta y_b}{2} - \sqrt{\frac{1}{4} + \left(\frac{\Delta y_b}{2}\right)^2},$$

where $\Delta y_b = y(hb, \ell b) - y(\ell b), \ell b)$. Moreover, $e_b^A < e_b^L$.

Comparing an achievement policy with laissez faire with regard to aggregate surplus, the policy is desirable ($S^A > S^L$) if

$$\pi e_p^L \frac{2y(\ell p, hp) - y(\ell p, \ell p) - y(hp, hp)}{2} + (1-\pi) e_u^L \frac{2y(\ell u, hu) - y(\ell u, \ell u) - y(hu, hu)}{2}$$
$$> \pi(e_p^L - e_p^A)(y(\ell p, hp) - y(\ell p, \ell p)) + (1-\pi)(e_u^L - e_u^A)(y(\ell u, hu) - y(\ell u, \ell u))$$
$$- \frac{(e_p^L)^2 - (e_p^A)^2 + (e_u^L)^2 - (e_u^A)^2}{2}. \quad (2)$$

This shows the same trade-off as condition (1): property (DD) implies that an achievement policy derives gains from resorting ($2(y(s,s') - y(s,s)) > y(s',s') - y(s,s)$ for $s' > s$), at the cost of depressing investment incentives ($e_b^A < e_b^L$). The first line of (2) captures these gains while the second reflects the decrease in investments net of savings in effort cost in the third line. Clearly as $y(hu,hu) - y(hu,\ell u)$ and $y(hp,hp) - y(hp,\ell p)$ approach zero, achievement based policies appear more attractive compared to the laissez faire outcome. That is, when gains from diversity are strong, static gains from re-matching may dominate adverse dynamic incentive effects.

Background Based AR

As argued above, a background based policy may avoid depressing investment incentives to some extent by conditioning the rematch on background, which is not subject to individual choice. A background based policy assigns individuals on the labor market based on their background, forming (u,p) firms whenever possible and using uniform rationing elsewhere.

Consider such policy for example in the case $\pi \leq 1/2$. The underprivileged obtain a privileged match with probability $\pi/(1-\pi)$, and an underprivileged match with probability $(1-2\pi)/(1-\pi)$; the privileged match with the underprivileged with certainty. Hence, there are measure π of (u,p) and $1/2 - \pi$ of (u,u) firms; the matching in educational attainment is subject to choice, however.

Therefore a privileged high achiever hp is assigned to an underprivileged high achiever hu at wage $y(hu,hp)/2$ with probability $\min\{e_u/e_p; 1\}$, and to an underprivileged low achiever ℓu at wage $y(\ell u, hp)/2$ otherwise. A privileged low achiever ℓp is matched to an underprivileged high achiever hu at wage $y(hu, \ell p)/2$ with probability $\max\{(e_u - e_p)/(1-e_p); 0\}$ and to an underprivileged low achiever ℓu at wage $y(\ell u, \ell p)/2$ otherwise.

An underprivileged high achiever obtains a hu match and wage $y(hu,hu)/2$ with probability $(1-2\pi)/(1-\pi)$, a hp match and wage $y(hu,hp)$ with probability $\min\{e_p/e_u; 1\}\pi/(1-\pi)$, and a ℓp match at wage $y(hu, \ell p)$ otherwise. An underprivileged low achiever gets a ℓu match and wage $y(\ell u, \ell u)/2$ with probability $(1-2\pi)/(1-\pi)$, a hp match and wage $y(\ell u, hp)/2$ with probability $\max\{(e_p - e_u)/(1-e_u); 0\}\pi/(1-\pi)$, and wage $y(\ell u, \ell p)/2$ otherwise.

Summarizing, a privileged agent's investment choice satisfies (see Appendix for details)

$$e_p^B = \frac{2\pi - 1}{\pi} e_p^L + \frac{1-\pi}{\pi} x_p \text{ if } \pi > 1/2 \text{ and } e_p^B = x_p \text{ otherwise,} \tag{3}$$

where

$$x_p = \frac{e_u^B}{e_p^B}\frac{y(hu, hp) - y(\ell u, hp)}{2} + \frac{y(\ell u, hp) - y(\ell u, \ell p)}{2} \qquad \text{if } e_p^B > e_u^B$$
$$x_p = \frac{1-e_u^B}{1-e_p^B}\frac{y(hu, \ell p) - y(\ell u, \ell p)}{2} + \frac{y(hu, hp) - y(hu, \ell p)}{2} \qquad \text{if } e_p^B < e_u^B.$$

Similarly, an underprivileged agent's investment is given by

$$e_u^B = x_u \text{ if } \pi > 1/2 \text{ and } e_u^B = \frac{1-2\pi}{1-\pi} e_u^{LF} + \frac{\pi}{1-\pi} x_u \text{ otherwise,} \tag{4}$$

where

$$x_u = \frac{y(hu, hp) - y(\ell u, hp)}{2} + \frac{1-e_p^B}{1-e_u^B}\frac{y(\ell u, hp) - y(\ell u, \ell p)}{2} \qquad \text{if } e_p^B > e_u^B$$
$$x_u = \frac{y(hu, \ell p) - y(\ell u, \ell p)}{2} + \frac{e_p^B}{e_u^B}\frac{y(hu, hp) - y(hu, \ell p)}{2} \qquad \text{if } e_p^B < e_u^B.$$

Using the notation defined above, aggregate surplus under a background based policy S^B can be written as

$$S^B = \pi \frac{(e_p^B)^2}{2} + (1-\pi)\frac{(e_u^B)^2}{2} + \begin{cases} \pi y(\ell u, \ell p) + (1-2\pi)\frac{y(\ell u, \ell u)}{2} & \text{if } \pi \le \frac{1}{2}, \\ (1-\pi) y(\ell u, \ell p) + (2\pi - 1)\frac{y(\ell p, \ell p)}{2} & \text{otherwise.} \end{cases}$$

That is, S^B depends on investment levels e_p^B and e_u^B and the quality of sorting on the labor market reflected by the measure of (u, p) firms and expressed by the third term. Any policy has to achieve the double goal of efficient (static) sorting and setting correct (dynamic) incentives. A background based policy is therefore clearly most successful at sorting the labor market efficiently when privileged and underprivileged are of even proportions ($\pi = 1/2$). Less obviously, in this case a background based policy does also very well in terms of incentive provision as stated in the following proposition.

Proposition 4 *When background types are of even proportions ($\pi = 1/2$), an equilibrium allocation under a background based policy coincides with an equilibrium allocation under fully transferable utility. If $y(\ell u, hp) = y(hu, \ell p)$ the TU equilibrium allocation is unique.*

That is, a background based policy indeed coincides with a benchmark allocation when the distribution of backgrounds in the population matches the optimal background intensity in production. This remains true when considering any finite number of background types. While the proof can be found in the appendix, an intuition follows. When $\pi = 1/2$ all agents choose their investment based on the certain knowledge that they will be matched into a team that is heterogeneous in backgrounds. Hence, investment incentives will be determined by the difference in individual payoffs in a (ℓ, ℓ) and a (h, h) team given that background is mixed. This difference coincides with the marginal social benefit of education under full transferable utility. This is due to property (M), which ensures that integration on the background dimension yields higher aggregate utility than full integration. Hence, there exists an equilibrium under fully transferable utility supported by market wages providing the same returns to education investment.

Compare now a background based policy to the laissez faire outcome in terms of aggregate surplus. Analyzing the case $\pi > 1/2$ in detail, independently of whether e_p^B is greater or lower than e_u^B the difference in total surplus between the two regimes is given by

$$\begin{aligned} S^B - S^L = {} & \pi \frac{(e_p^B)^2 - (e_p^L)^2}{2} + (1-\pi)\frac{(e_u^B)^2 - (e_u^L)^2}{2} - (1-\pi)e_u^B \\ & + (1-\pi)\left(\frac{y(hu, hp) - y(\ell u, \ell p)}{2} + y(\ell u, \ell p) - \frac{y(\ell p, \ell p) + y(\ell u, \ell u)}{2}\right). \end{aligned}$$

Note that the difference $(1-\pi)\left((y(hu, hp) - y(\ell u, \ell p))/2 - e_u^B\right) > 0$ reflects the mismatch between social and private incentives under a background based policy.[11] This mismatch is 0 when $e_p = e_u$ for $\pi = 1/2$ and increase in π. As property (DD) implies $2y(\ell u, \ell p) > y(\ell u, \ell u) + y(\ell p, \ell p)$, a sufficient condition for a background based policy to dominate laissez faire is

$$(1-\pi)\left({e_u^B}^2 - {e_u^L}^2\right) > \pi\left({e_p^L}^2 - {e_p^B}^2\right).$$

That is, if investments are higher if firms are integrated in background by a policy than if firms remain segregated under laissez faire, a background based policy appears desirable from an aggregate surplus point of view.

[11] To see this note that $\frac{y(hu,hp)-y(\ell u,\ell p)}{2} - e_u^B = \frac{y(hp,\ell u)-y(\ell u,\ell p)}{2}\frac{e_p^B - e_u^B}{1-e_u^B}$.

Indeed whenever the return to education in firms that are integrated by background ($y(hu, hp) - y(\ell u, \ell p)$) if sufficiently great, a background based policy achieves higher aggregate surplus than the laissez-faire allocation. The proof can be found in the appendix.

Proposition 5 *If $y(hu, hp) - y(\ell u, \ell p)$ is sufficiently high, $S^B > S^L$ for all $0 < \pi < 1$ and $S^B = S^L$ for $\pi = 0$ and $\pi = 1$.*

An interesting case occurs when $e_u^L = y(hu, hu) - y(\ell u, \ell u) < y(hu, hp) - y(\ell u, \ell p) < y(hp, hp) - y(\ell p, \ell p) = e_p^L$. Then a background based policy encourages the underprivileged to invest while discouraging the privileged, since segregation payoffs for high achievers, which are high for the privileged, but low for the underprivileged, are no longer attained with certainty. This reflects the encouragement effect of affirmative action discussed by Coate and Loury (1993), inducing the underprivileged to invest more now that they expect a significant return because of the policy. Note though, that here, as elsewhere, the privileged agents' incentives are reduced. Indeed this is a general insight from our analysis: the group not favored by the policy has reduced incentives; the group favored by the policy may have improved incentives, as in this case, or not, as in the case of achievement based policy.

Background versus Achievement Based Policies

Finally, note that an achievement based policy might be preferable in terms of aggregate surplus to one based on background despite the adverse effects on incentives. Since a background based policy induces outcomes that are equivalent to laissez-faire when $\pi = 0$ or $\pi = 1$, an achievement based policy may dominate both in the neighborhood of the extremes if gains from rematching outweigh the losses from depressed investments. We have $S^B > S^A$ if

$$\pi \frac{(e_p^B)^2}{2} + (1-\pi)\frac{(e_u^B)^2}{2} + \min\{\pi; 1-\pi\}\frac{2y(\ell u, \ell p) - y(\ell u, \ell u) - y(\ell p, \ell p)}{2}$$
$$> \pi e_p^A \left(y(hp, \ell p) - y(\ell p, \ell p) - \frac{e_p^A}{2} \right) + (1-\pi) e_u^A \left(y(\ell u, hu) - y(\ell u, \ell u) - \frac{e_u^A}{2} \right).$$

To illustrate suppose that $\pi \leq 1/2$ and $e_p^B \geq e_u^B$. Then $S^B > S^A$ if

$$\begin{aligned}
&(e_p^B - e_p^A)(y(\ell u, hp) - y(\ell u, \ell p)) + (e_u^B - e_u^A)(y(hu, hp) - y(\ell u, hp)) \\
&\qquad + e_p^A(y(\ell u, hp) - y(\ell u, \ell p) - y(\ell p, hp) + y(\ell p, \ell p)) \\
&\qquad + e_u^A(y(hu, hp) - y(\ell u, hp) - y(\ell u, hu) + y(\ell u, \ell u)) \\
&> \frac{(e_p^B)^2 + (e_u^B)^2}{2} - \frac{(e_p^A)^2 + (e_u^A)^2}{2}.
\end{aligned}$$

Here the first line captures gains from higher investment incentives under a background policy, the second and third, gains (or losses) from sorting by background versus achievement, and the fourth reflects effort cost savings under an achievement based policy. Property (M) implies $y(hu, hp) - y(\ell u, \ell p) > y(\ell p, hp) - y(\ell u, \ell p) + y(hu, \ell u) - y(\ell u, \ell p)$, and therefore $S^B > S^A$ and $S^B > S^L$ for $\pi = 1/2$. For polar values of π (i.e., $\pi = 0$ or $\pi = 1$) the society is homogenous in background, and therefore a background based policy coincides with laissez-faire. Applying the results of Section 2, an achievement based policy dominates both regimes when gains from diversity and thus the benefits from resorting are great. This and the fact that payoffs under laissez-faire and both policies are continuous in π at $\pi = 0$ and $\pi = 1/2$ imply the following proposition.

Proposition 6 *If the economy is sufficiently homogenous in background b (*π *sufficiently close to* 0 *or* 1*) an achievement based policy dominates both a background based policy and laissez faire if gains from diversity in achievement are sufficiently great (*$y(hb, hb) - y(hb, \ell b)$ *sufficiently close to* 0*). If the economy is sufficiently diverse in background (*π *close to* $1/2$*), a background based policy dominates both an achievement based policy and laissez-faire.*

The second statement follows from Proposition 4 and the fact that investments e_p^B and e_u^B converge to $y(hu, hp) - y(\ell u, \ell p)$ as π approaches $1/2$.

Summarizing, we find that when segregation in background discourages education investment (as $y(hu, hu) - y(\ell u, \ell u)$ is small), some policy is desirable if gains from diversity in background ($y(hu, hp) - y(\ell u, \ell p)$) are great. A background based policy is always desirable when the economy is diverse in backgrounds (π close to $1/2$), and an achievement based policy may be desirable in homogenous economies when diversity in achievements is highly desirable.

3.4 Characterization When the Underprivileged Cannot Invest

We consider here the special case where only the privileged can invest. There is a measure $1-\pi$ of underprivileged agents who are always low achievers at the time the labor market opens.

The analysis of section 2 can be replicated simply with the caveat that for consistency the measure q of high achievers must now be equal to πe, where e is the privileged agents' effort. In particular, Lemma 1 and its corollary generalizes to

Lemma 2 *Let π be the measure of agents who are able to invest. Suppose that utility is fully transferable.*

(i) If $W-w>\frac{1}{4\pi}$, $e^T=2(W-w)$ and $q>1/2$,

(ii) If $W-w<\frac{1}{4\pi}<\min\{w;1/2\}$, $e^T=\frac{1}{2\pi}$ and $q=1/2$,

(iii) If $\min\{w;1/2\}<\frac{1}{4\pi}$, $e^T=\min\{2w;1\}$ and $q<1/2$.

Corollary 3 *Comparing investment levels when utility is perfectly transferable (TU) and strictly nontransferable (LF) yields*

$$e^L>e^T\Leftrightarrow W>\frac{1}{2\pi}.$$

The analysis of achievement based policies follows that in section 2. In particular, since when $\pi=1$ the measure of high achievers is less than 1/2, this is also the case when there is a smaller measure of agents who can invest. It is then simple to show that the total surplus under an achievement based policy is

$$S^A=\pi e^A\left(2w-\frac{e^A}{2}\right).$$

where

$$e^A=w-\frac{1}{2\pi}\left(\sqrt{4\pi^2w^2+1}-1\right)$$

e^A decreases with π but the measure of high achievers $q=\pi e^A$ increases in π.

There is again a tradeoff between improving the quality of the match via the achievement policy versus depressing investment incentives. Because total surplus is now a function of π, the cutoff value of W for which the policy is surplus equivalent to laissez-faire is also a function of π.

Corollary 4 *Total surplus under an achievement based policy is higher than under laissez-faire if and only if $W \leq W_0(w, \pi)$, where $W_0(w, \pi)$.*

The cutoff satisfies $W_0(w, \pi) \in [0, 2w]$, increases in w and decreases in π. Indeed, the larger is π, the more likely an ℓ agent will get w rather than 0, so investment incentives are weakened from this insurance effect.

Consider now a background based policy.

In case $\pi \leq 1/2$ each u agent is assigned a p match with probability $\pi/(1-\pi)$ and u otherwise, while p agents obtain a u match with certainty. As u agents attain ℓ under segregation at school, a p agent with high achievement is matched into an integrated firm (h, ℓ) obtaining wage w for sure, as under an achievement based policy. Privileged low achievers, however, have now probability 0 of matching into a (h, ℓ) firm. This reduces their expected payoff compared to an achievement based policy. Hence, when $\pi < 1/2$, a background based policy induces a redistribution towards u agents (having a higher chance to obtain wage w) and stronger incentives for p agents. As the measure of firms (h, ℓ) is $\pi e^B > \pi e^A$, a background based policy increases total output and dominates an achievement based policy when $\pi \leq 1/2$.

Hence, it will generate higher aggregate surplus than laissez-faire also in the neighborhood of the curve $W_0(w, \pi)$. As above laissez-faire induces better incentives, but less efficient matches. One can show that there is a cutoff value $W_2(w, \pi) > W_0(w, \pi)$, such that laissez-faire yields higher total surplus than a background based policy if and only if $W > W_2(w, \pi)$. For $\pi \leq 1/2$, $W_2(w, \pi) = \sqrt{3}w$.

When $\pi > 1/2$, a privileged agent optimally invests

$$e^B = \underbrace{\frac{1-\pi}{\pi}w}_{\text{match with } u \text{ agent}} + \underbrace{\frac{2\pi-1}{\pi}W}_{\text{match with a high achieving } p \text{ agent}} . \tag{5}$$

As $e^A < w$ we have $e^B > e^A$, and a background based policy induces redistribution towards the underprivileged and stronger incentives for the privileged as in the case above.

Yet now total surplus need not be higher under a background based policy. As $\pi > 1/2$, some p agents must form (p, p) matches. Since a background based policy does not condition on achievement, these agents segregate as under laissez-faire. Hence, there is a positive measure of (h, h) and (ℓ, ℓ) firms, which is inefficient from a surplus point of view. That is, for $\pi > 1/2$ a background based policy induces better incentives and a less efficient matching than an achievement based policy. An argument similar to the one in Corollary 2 yields a cutoff $W_1(w, \pi)$ such that a background based policy is preferable to one based on achievement if $W > W_1$, and the reverse is true if $W < W_1$. Since the amount of mismatch increases in π, so does $W_1(w, \pi)$, which also increases in w.

When compared to laissez-faire in case $\pi > 1/2$ a background based policy provides worse incentives but a more efficient matching, as above. The following proposition gives the surplus maximizing policy depending on parameters, and is illustrated in Figure 1.

Proposition 7 *The surplus maximizing policy is*

(i) Laissez-faire when $W \geq W_2(w, \pi)$,

(ii) Background based when $W \in (W_2(w, \pi), W_1(w, \pi))$, *and*

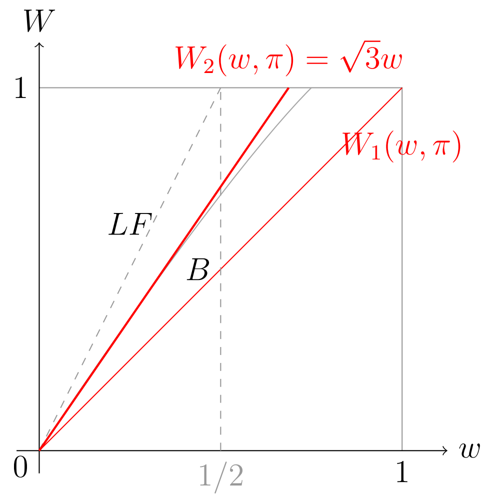
(iii) Achievement based when $W < W_1(w, \pi)$.

(iv) Background policies dominate achievement policies when $\pi < 1/2$.

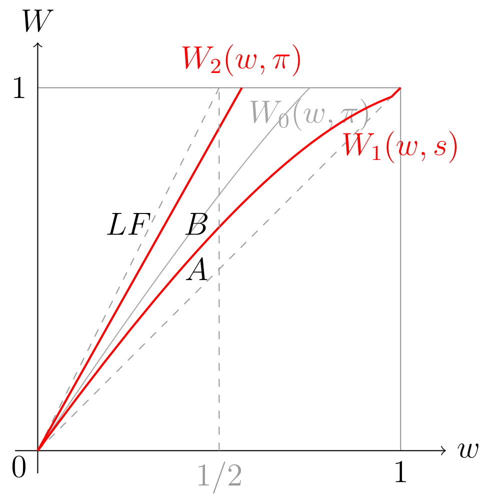
In the general analysis, condition (M) requires a strict inequality, while there is an equality when $lu = hu = lp$. While we lose some of our previous results, the special case illustrates the tradeoffs between achievement and background based policies in a straightforward way. Note that for $\pi = 1/2$ a background based policy dominates an achievement based policy, which is consistent with the result in Proposition 6. More generally, this case provides a simple illustration of the roles of background homogeneity in the population (π close to zero or to one) and the intensity of the diversity benefit (as measured by $W - w$) for policy implications. For instance, as the diversity benefit is small ($W - w$ small), a background based policies does well in populations with a large proportion of underprivileged, but is dominated by an achievement based policy when the proportion of privileged is large.

4 Conclusion

We presented a framework to analyze policies of associational redistribution on the labor market and at school. The framework imposes strictly



(a) $\pi < 1/2$



(b) $\pi > 1/2$

Figure 1: Laissez-faire, Achievement, and Background Based Policies

nontransferable utility serving to focus on the interaction of matching patterns and investment incentives. It remains silent, however, about another source of inefficiencies when utility is transferable, but not perfectly so. Then competition may require inefficient sharing of surplus (see e.g., Legros and Newman, 2008) which in turn affects investment incentives. Pursuing this topic appears to be an important task for future research.

In the present approach policies aim at replicating the fully transferable utility matching outcome, that is integration, as a benchmark. In a more complex derivation of nontransferable utility, nontransferabilities may affect the optimal matching, however. See Gall et al. (2008) for an example when information rents decrease in the scope of the project, so that the optimal matching involves integration when there is asymmetric information generating nontransferabilities, but segregation under perfect information.

Labor market policies need to trade off output efficient sorting and provision of adequate incentives for pre-match investments. Conditioning labor market re-matching on observable information not subject to individual choice, such as background, appears beneficial when it is linked to education outcome. Our framework enables evaluation of different policies of associational redistribution. Performance of policies typically depend on characteristics of the economy. For instance, if the returns to education depend on the firm composition such that integration boosts returns of underprivileged workers, a labor market wages under laissez faire fail to reflect this and some policy almost always dominates laissez faire. In a balanced economy, where measures of underprivileged and privileged agents match, a background based policy achieves the fully transferable utility benchmark. In polarized economies where one type of background heavily outweighs the other, an achievement based policy may in fact be desirable if gains from diversity in achievement are sufficiently strong.

A Mathematical Appendix

Proof of Lemma 1

When maximizing expected utility $u = ew(h) + (1-e)w(\ell) - \frac{e^2}{2}$, a necessary condition for optimal investment is $e^{TU} = 2(w(h) - w)$.

We have established in the text that if $q > 1/2$, agents with education h are abundant and obtain wage $w(h) = W$, agents with ℓ obtain $w(\ell) = 2w - W$. Hence, $e^{TU} = 2(W - w)$ and the realized $q = 2(W - w)$, which is greater than 1/2 only if $W - w > 1/4$.

If $q < 1/2$, h agents are scarce, so that $w(h) = 2w$ and $w(\ell) = 0$. As $e^{TU} = \min\{2w; 1\}$, $q = e^{TU} < 1/2$ only if $\min\{2w; 1\} < 1/2$.

Finally, if $q = 1/2$ a continuum of wages is consistent with a stable allocation: $w(h) \in [W, 2w]$ and $w(\ell) = 2w - w(h)$. Agents choose $e^{TU} = 2(w(h) - w)$. $q = 2(w(h) - w)$ is equal to 1/2 only if $w(h) = w + 1((4)$. Therefore $e^{TU} = 1/(2)$ in this case.

Suppose first $q = 2(w(h) - w) > 1/2$. This is only consistent with $2(W - w) > 1/2$. $q < 1/2$ is only consistent with $\min\{2w; 1\} < 1/2$. For intermediate cases, that is $(W - w) < 1/4 < \min\{w; 1/2\}$, $q = 1/2$. Therefore $e = 2w(h) - 2w = 1/(2)$, that is $w(h) = w + 1/4$.

Proof of Corollary 1

When $W - w > 1/(4\pi)$, the property DD ($2w > W$) implies that $W > 1/(2\pi)$. In this case, $e^{LF} - e^{TU} = 2w - W > 0$.

When $\min\{w; 1/1\} < 1/(4\pi)$, $W < 1/(2\pi)$ since $2w > W$ and $W < 1$. In this case, $e^{LF} - e^{TU} = W - 2w < 0$.

In the intermediate case $W - w < 1/4 < \min\{w; 1/2\}$, $e^{TU} = 1/2$ and therefore $e^{LF} - e^{TU}$ is positive only if $W > 1/2$.

Proof of Proposition 1

Given an AR policy which assigns h agents to ℓ agents whenever possible, an agent chooses effort e to solve

$$\begin{aligned} &\max\nolimits_e \quad e\left(\frac{1-q}{q}w + \frac{2q-1}{q}W\right) + (1-e)w - \frac{e^2}{2} \text{ if } q > 1/2, \\ &\max\nolimits_e \quad ew + (1-e)\frac{q}{1-q}w - \frac{e^2}{2} \text{ if } q \leq 1/2. \end{aligned} \tag{6}$$

Supposing $q > 1/2$, a necessary condition for investment is

$$e = \frac{2q-1}{q}(W - w).$$

In equilibrium $e = q$ must hold, and this equality implies

$$q^2 - 2(W-w)q + (W-w) = 0$$

but the discriminant is $(W-w)^2 - (W-w) = (W-w)(W-w-1) < 0$ since $W < 1$. Therefore in any equilibrium $q \leq 1/2$ and

$$e^A = \frac{1-2q}{1-q}w < w. \tag{7}$$

Replacing e^A by q and solving for q yields the expression in the proposition (the other solution is greater than 1). Clearly, the solution is less than 1/2. Since $e^A < w$ both $e^A < e^{LF}$ and $e^A < e^{TU}$. Under an achievement based policy investments satisfy (7). With $q = e^A$

$$e^A = w + \frac{1}{2} - \sqrt{w^2 + \frac{1}{4}}.$$

Proof of Corollary 2

The condition $S^A > S^{LF}$ holds, if e^A solves the quadratic equation ${e^A}^2 - 4we^A + W^2 < 0$. Solving yields

$$e^A > 2w - \sqrt{4w^2 - W^2}. \tag{8}$$

Since $e^A < w$, $W^2 \geq 3w^2$ implies $S^A < S^{LF}$, that is $W_0 < \sqrt{3}w$. Finally, using (1), $W_0(w, \pi)$ solves

$$W(2w - W) = (W - e^A)2w - \frac{1}{2}(W^2 - {e^A}^2) \tag{9}$$

By Proposition 1, differentiating q with respect to w, and using $q = e^A$, e^A is an increasing function of w. Therefore the RHS of (9) decreases in w, and increases in W since $W < 1$. The LHS increases in w and decreases in W since $w < W$. Hence as w increases, so must W to restore equality. Hence, $W_0(w)$ increases in w.

Table 1: Equilibrium Regimes Under Full TU

π	condition	firms with positive measure				$e_p^{TU} = w(hp) - w(\ell p)$	$e_u^{TU} = w(hu) - w(\ell u)$
$\pi > 1/2$	$\pi e_p > 1/2$	(hp, hp)	$(\ell p, hp)$	(hu, hp)	$(\ell u, hp)$	$y(hp, hp) - y(\ell p, hp)$	$y(hu, hp) - y(\ell u, hp)$
	$1/2 > \pi e_p > 1/2 - (1-\pi)(1-e_u)$	$(\ell p, hp)$	(hu, hp)	$(\ell u, hp)$	$(\ell u, \ell p)$	$y(\ell u, hp) - y(\ell u, \ell p)$	$y(hu, hp) - y(\ell u, hp)$
	$\pi e_p < 1/2 - (1-\pi)(1-e_u)$	$(\ell p, hp)$	(hu, hp)	$(hu, \ell p)$	$(\ell u, \ell p)$	$y(hu, hp) - y(hu, \ell p)$	$y(hu, \ell p) - y(\ell u, \ell p)$
	$\pi(1-e_p) > 1/2$	$(\ell p, hp)$	$(\ell p, \ell p)$	$(hu, \ell p)$	$(\ell u, \ell p)$	$y(\ell p, hp) - y(\ell p, \ell p)$	$y(hu, \ell p) - y(\ell u, \ell p)$
$\pi < 1/2$	$(1-\pi)e_u > 1/2$	(hu, hp)	$(hu, \ell p)$	(hu, hu)	$(\ell u, hu)$	$y(hu, hp) - y(hu, \ell p)$	$y(hu, hu) - y(\ell u, hu)$
	$\pi e_p > 1/2 - (1-\pi)(1-e_u)$	(hu, hp)	$(hu, \ell p)$	$(\ell u, hu)$	$(\ell u, hp)$	$y(hu, hp) - y(hu, \ell p)$	$y(hu, hp) - y(\ell u, hp)$
	$\pi e_p < 1/2 - (1-\pi)(1-e_u)$	(hu, hp)	$(hu, \ell p)$	$(\ell u, hu)$	$(\ell u, \ell p)$	$y(hu, hp) - y(hu, \ell p)$	$y(hu, \ell p) - y(\ell u, \ell p)$
	$(1-\pi)(1-e_u) > 1/2$	$(\ell u, hp)$	$(\ell u, \ell p)$	$(\ell u, hu)$	$(\ell u, \ell u)$	$y(\ell u, hp) - y(\ell u, \ell p)$	$y(\ell u, hu) - y(\ell u, \ell u)$

Proof of Proposition 3

Supposing that $e_b \geq 1/2$ for $b = u$ or $b = p$ is quickly led to a contradiction as follows. Individual i of background b chooses effort e_i to solve

$$\max_{e_i} e_i \left(\frac{1-e_b}{e_b} w(s(h,b), s(\ell,b)) + \frac{2e_b - 1}{e_b} w(s(h,b), s') \right) + (1-e_i) w(s(h,b), s(\ell,b)) - \frac{e_i^2}{2},$$

where $w(s(h,b), s(\ell,b)) = y(s(h,b), s(\ell,b))/2$ is the wage in a (h,b), (ℓ,b) firm and $w(s(h,b), s') \in \{y(s(h,b), s(h,b))/2; y(s(h,b), s(\ell,b'))/2\} > w(s(h,b), s(\ell,b))$, $b' \neq b$, is the wage in the firm a high achiever with background b is matched when not rationed to a low achiever of background b. A necessary condition for i's effort choice is

$$e_i = \frac{2e_b - 1}{e_b} [w(s(h,b), s') - w(s(h,b), s(\ell,b))].$$

Since $e_i = e_b$ in equilibrium and $w(s(h,b), s') - w(s(h,b), s(\ell,b)) \leq 1$ as $y(s(h,p), s(h,p)) < 2$ by property (DC), this equation cannot hold.

Hence, $e_u < 1/2$ and $e_p < 1/2$. By the monotonicity of $y(.)$, $y(\ell p, \ell p) > y(\ell u, \ell p)$, so that remaining ℓ agents segregate along backgrounds and form $(\ell u, \ell u)$ and $(\ell p, \ell p)$ firms. An agent of background b therefore solves

$$\max_{e_i} e_i \frac{1-2e_b}{1-e_b} \frac{y(s(h,b), s(\ell,b)) - y(s(\ell,b), s(\ell,b))}{2} + \frac{e_b}{1-e_b} \frac{y(s(h,b), s(\ell,b))}{2} + \frac{1-2e_b}{1-e_b} \frac{y(s(\ell,b), s(\ell,b))}{2} - \frac{e_i^2}{2}.$$

The necessary condition gives the expression in the proposition; it can be easily verified that the solution $e_b \in (0, 1/2)$. The last statement follows from the fact that $e_b < (y(s(h,b), s(\ell,b)) - y(s(\ell,b), s(\ell,b)))/2 < (y(s(h,b), s(h,b)) - y(s(\ell,b), s(\ell,b)))/2$ by monotonicity of $y(.)$. □

Derivation of e_p^B, e_u^B, and S^B

Let $\pi > 1/2$. Then a p agent has a chance of $(2\pi - 1)/\pi$ of matching with a p agent. Otherwise he matches with a u agent, and, if own achievement is h, obtains a h match with probability $\min\{e_u/e_p; 1\}$, and with probability $\max\{(e_u - e_p)/(1 - e_p); 0\}$ if own achievement is ℓ. Hence, a p agent's effort choice solves

$$\max_e e\left(\frac{2\pi-1}{\pi}\frac{y(hp,hp)}{2}+\frac{1-\pi}{\pi}\left(\min\left\{\frac{e_u}{e_p};1\right\}\frac{y(hu,hp)-y(\ell u,hp)}{2}+\frac{y(\ell u,hp)}{2}\right)\right)$$
$$+(1-e)\left(\frac{2\pi-1}{\pi}\frac{y(\ell p,\ell p)}{2}+\frac{1-\pi}{\pi}\left(\max\left\{\frac{e_u-e_p}{1-e_p};0\right\}\frac{y(hu,\ell p)-y(\ell u,\ell p)}{2}+\frac{y(\ell u,\ell p)}{2}\right)\right)$$
$$-\frac{e^2}{2}.$$

A u agent solves

$$\max_e e\left(\min\left\{\frac{e_p}{e_u};1\right\}\frac{y(hu,hp)-y(hu,\ell p)}{2}+\frac{y(hu,\ell p)}{2}\right)$$
$$+(1-e)\left(\max\left\{\frac{e_p-e_u}{1-e_u};0\right\}\frac{y(\ell u,hp)-y(\ell u,\ell p)}{2}+\frac{y(\ell u,\ell p)}{2}\right)-\frac{e^2}{2}.$$

Suppose $e_p > e_u$. Then

$$e_p = \frac{2\pi-1}{\pi}\frac{y(hp,hp)-y(\ell p,\ell p)}{2}$$
$$+\frac{1-\pi}{\pi}\left(\frac{e_u}{e_p}\frac{y(hu,hp)-y(\ell u,hp)}{2}+\frac{y(\ell u,hp)-y(\ell u,\ell p)}{2}\right),$$
$$e_u = \frac{y(hu,hp)-y(\ell u,\ell p)}{2}-\frac{e_p-e_u}{1-e_u}\frac{y(\ell u,hp)-y(\ell u,\ell p)}{2}.$$

Suppose $e_p < e_u$. Then

$$e_p = \frac{2\pi-1}{\pi}\frac{y(hp,hp)-y(\ell p,\ell p)}{2}$$
$$+\frac{1-\pi}{\pi}\left(\frac{y(hu,hp)-y(\ell u,\ell p)}{2}-\frac{e_u-e_p}{1-e_p}\frac{y(hu,\ell p)-y(\ell u,\ell p)}{2}\right),$$
$$e_u = \frac{y(hu,\ell p)-y(\ell u,\ell p)}{2}+\frac{e_p}{e_u}\frac{y(hu,hp)-y(hu,\ell p)}{2}.$$

Note that investment incentives of p and u agents can only be aligned if $\pi = 1/2$. Therefore $e_p \neq e_u$ for $\pi \neq 1/2$. Both a regime where p agents invest a lot and u agents a little, and the reverse may emerge in equilibrium, both are possible for π sufficiently close to 1/2.

Let now $\pi \leq 1/2$. Then a p agent matches with a u agent with certainty, and, if own achievement is h, obtains a h match with probability $\min\{e_u/e_p;1\}$, and with probability $\max\{(e_u-e_p)/(1-e_p);0\}$ if own achievement is ℓ. Hence, a p agent solves

$$\max_e e\left(\min\left\{\frac{e_u}{e_p};1\right\}\frac{y(hu,hp)-y(\ell u,hp)}{2}+\frac{y(\ell u,hp)}{2}\right)$$
$$+(1-e)\left(\max\left\{\frac{e_u-e_p}{1-e_p};0\right\}\frac{y(hu,\ell p)-y(\ell u,\ell p)}{2}+\frac{y(\ell u,\ell p)}{2}\right)-\frac{e^2}{2}.$$

A u agent is assigned to a p agent with probability $\pi/(1-\pi)$ and otherwise matches with a u agent. Therefore a u agent solves

$$\max_e e\left(\frac{1-2\pi}{1-\pi}\frac{y(hu,hu)}{2}+\frac{\pi}{1-\pi}\left(\min\left\{\frac{e_p}{e_u};1\right\}\frac{y(hu,hp)-y(hu,\ell p)}{2}+\frac{y(hu,\ell p)}{2}\right)\right)$$
$$+(1-e)\left(\frac{1-2\pi}{1-\pi}\frac{y(\ell u,\ell u)}{2}+\frac{\pi}{1-\pi}\left(\max\left\{\frac{e_p-e_u}{1-e_u};0\right\}\frac{y(\ell u,hp)-y(\ell u,\ell p)}{2}+\frac{y(\ell u,\ell p)}{2}\right)\right)-\frac{e^2}{2}.$$

First order conditions imply that $e_p \neq e_u$ for $\pi \neq 1/2$. Supposing $e_p > e_u$

$$e_p = \frac{y(\ell u,hp)-y(\ell u,\ell p)}{2}+\frac{e_u}{e_p}\frac{y(hu,hp)-y(\ell u,hp)}{2},$$
$$e_u = \frac{1-2\pi}{1-\pi}\frac{y(hu,hu)-y(\ell u,\ell u)}{2}+\frac{\pi}{1-\pi}\left(\frac{y(hu,hp)-y(\ell u,\ell p)}{2}-\frac{e_p-e_u}{1-e_u}\frac{y(\ell u,hp)-y(\ell u,\ell p)}{2}\right).$$

Otherwise, if $e_p < e_u$

$$e_p = \frac{y(hu,hp)-y(\ell u,\ell p)}{2}-\frac{e_u-e_p}{1-e_p}\frac{y(hu,\ell p)-y(\ell u,\ell p)}{2},$$
$$e_u = \frac{1-2\pi}{1-\pi}\frac{y(hu,hu)-y(\ell u,\ell u)}{2}+\frac{\pi}{1-\pi}\left(\frac{y(hu,\ell p)-y(\ell u,\ell p)}{2}+\frac{e_p}{e_u}\frac{y(hu,hp)-y(hu,\ell p)}{2}\right).$$

Again both regimes are possible for π sufficiently close to 1/2.

Suppose that $e_p^B > e_u^B$, then

$$\begin{aligned}S^B =& e_u^B(1-\pi)(y(hu,hp)-y(\ell u,hp))+(1-\pi)y(\ell u,\ell p)+(2\pi-1)\frac{y(\ell p,\ell p)}{2}\\ &+e_p^B\left[(1-\pi)(y(\ell u,hp)-y(\ell u,\ell p))+(2\pi-1)\frac{y(hp,hp)-y(\ell p,\ell p)}{2}\right]\\ &-\frac{(1-\pi)(e_u^B)^2+\pi(e_p^B)^2}{2},\end{aligned}$$

if $\pi > 1/2$ and

$$\begin{aligned}S^B =& e_p^B\pi(y(\ell u,hp)-y(\ell u,\ell p))+\pi y(\ell u,\ell p)+(1-2\pi)\frac{y(\ell u,\ell u)}{2}\\ &+e_u^B\left[\pi(y(hu,hp)-y(\ell u,hp))+(1-2\pi)\frac{y(hu,hu)-y(\ell u,\ell u)}{2}\right]\\ &-\frac{(1-\pi)(e_u^B)^2+\pi(e_p^B)^2}{2}.\end{aligned}$$

if $\pi \leq 1/2$. The cases $e_u^B > e_p^B$ are analogous.

Proof of Proposition 4

Note first that for $\pi = 1/2$ $e_p^B = e_u^B = (y(hu, hp) - y(\ell u, \ell p))/2$ give equilibrium investment levels under a background based policy. Total surplus is then given by

$$\begin{aligned} S^B =& \pi e_p^B \frac{y(hu,hp) - y(\ell u, \ell p)}{2} + (1-\pi) e_u^B \frac{y(hu,hp) - y(\ell u, \ell p)}{2} + \frac{y(\ell u, \ell p)}{2} \\ & - \pi \frac{(e_p^B)^2}{2} - (1-\pi)\frac{(e_u^B)^2}{2}. \end{aligned}$$

Clearly $e_p^B = e_u^B = (y(hu, hp) - y(\ell u, \ell p))/2$ solves the social planner's problem given that $e_p^B = e_u^B$. To check whether this coincides with the TU outcome, note that three cases are possible when $\pi = 1/2$. $e_u^T > e_p^T$, $e_u^T < e_p^T$, or $e_u^T = e_p^T$. Assuming the first, measure $e_p^T/2$ of (hp, hu) firms, measure $(e_u^T - e_p^T)/2$ of $(hu, \ell p)$ and measure $(1 - e_u^T)/2$ of $(\ell u, \ell p)$ firms emerge. Wages satisfy

$$w(hp) = y(hu, hp) - w(hu), w(hu) = y(hu, \ell p) - w(\ell p), w(\ell p) = y(\ell u, \ell p) - w(\ell u).$$

This yields investments

$$\begin{aligned} e_p^T &= w(hp) - w(\ell p) = y(hu, hp) - y(hu, \ell p), \\ e_u^T &= w(hu) - w(\ell u) = y(hu, \ell p) - y(\ell u, \ell p). \end{aligned}$$

Consistency requires $e_p^T < e_u^T$, that is $y(hu, hp) - y(hu, \ell p) < y(hu, \ell p) - y(\ell u, \ell p)$. Similarly, in the case $e_u < e_p$ consistency requires that $y(\ell u, hp) - y(\ell u, \ell p) > y(hu, hp) - y(\ell u, hp)$. In case $e_u = e_p$, the wage profile is not uniquely determined by scarcity, and consistency is satisfied for any wage profile $w(hp) = w(hu)$ and $w(\ell p) = w(\ell u)$ independent of the technology. Therefore the wage profile $w(hp) - w(\ell p) = w(hu) - w(\ell u) = (y(hu, hp) - y(\ell u, \ell p))/2$ is always an equilibrium outcome under full TU. In case $y(\ell u, hp) = y(hu, \ell p)$ both inequalities required for consistency of asymmetric outcomes coincide and contradict property (M). This establishes the proposition.

Proof of Proposition 5

Note first that $\pi {e_p^B}^2 + (1-\pi){e_u^B}^2 > \pi {e_p^L}^2 + (1-\pi){e_u^L}^2$ is implied by $y(hu, hp) - y(\ell u, \ell p)$ sufficiently high compared to e_p^L and e_u^B. For $\pi \geq 1/2$, $y(hu, hp) - y(\ell u, \ell p) > y(hp, hp) - y(\ell p, \ell p)$ implies $e_p^B \geq e_p^L$ since e_p^B has a minimum at

e_p^L, and $e_u^B > e_u^L$ which can be verified by setting $e_p^B = e_p^L$ in the expression for x_u and noting that $e_p^L > e_u^L$. For $\pi \leq 1/2$, property (DC) ensures that $y(hu, hp) - y(\ell u, \ell p) > y(hu, hu) - y(\ell u, \ell u)$, which implies in turn that $e_u^B \geq e_u^L$ attaining the minimum e_u^L at $\pi = 0$. If $y(hu, hp) - y(\ell u, \ell p)$ great enough, $e_p^B \geq e_p^L$ and $\pi {e_p^B}^2 + (1-\pi){e_u^B}^2 > \pi {e_p^L}^2 + (1-\pi){e_u^L}^2$.

Suppose now that $y(hu, hp) - y(\ell u, \ell p) < y(hp, hp) - y(\ell p, \ell p)$, that is $e_p^B < e_p^L$ for $\pi < 1$. Differentiating $S^B - S^L$ with respect to π yields

$$\frac{\partial(S^B - S^L)}{\partial \pi} = -\frac{(e_p^L)^2 - (e_p^B)^2}{2} - \frac{(e_u^B)^2 - (e_u^L)^2}{2} - (1-\pi)(1-e_u^B)\frac{\partial e_u^B}{\partial \pi} + \pi e_p^B \frac{\partial e_p^B}{\partial \pi} - \left(\frac{y(hu, hp) + y(\ell u, \ell p)}{2} - e_u^B - \frac{y(\ell p, \ell p) + y(\ell u, \ell u)}{2}\right).$$

Note that $S^B - S^L$ has a zero at $\pi = 1$, as does its derivative. Indeed for all $\pi \in [1/2, 1]$, whenever $S^B = S^L$, then

$$\frac{\partial(S^B - S^L)}{\partial \pi} = -\frac{1}{1-\pi}\frac{(e_p^L)^2 - (e_p^B)^2}{2} - (1-\pi)(1-e_u^B)\frac{\partial e_u^B}{\partial \pi} + \pi e_p^B \frac{\partial e_p^B}{\partial \pi}.$$

Hence, $\frac{\partial(S^B - S^L)}{\partial \pi} < 0$ at a zero whenever

$$\frac{(e_p^L)^2 - (e_p^B)^2}{2} > (1-\pi)\left(\pi e_p^B - (1-\pi)(1-e_u^B)\frac{\partial e_u^B}{\partial e_p^B}\right)\frac{\partial e_p^B}{\partial \pi} \tag{10}$$

$$\Leftrightarrow \frac{e_p^L + e_p^B}{2} > \frac{e_p^B\left(\pi e_p^B - (1-\pi)(1-e_u^B)\frac{\partial e_u^B}{\partial e_p^B}\right)}{\pi e_p^B + (1-\pi)A\left(\frac{e_u^B}{e_p^B} - \frac{\partial e_u^B}{\partial e_p^B}\right)}$$

where

$$A = \frac{y(hu, hp) - y(\ell u, hp)}{2}.$$

Using the definition of e_p^B for $\pi > 1/2$ from (3) the condition becomes

$$\frac{e_p^L - B}{2} + A\frac{e_p^L e_u^B}{2(e_p^B)^2} > -\left(1 - e_u^B - \frac{A}{2}\left(1 + \frac{e_p^L}{e_p^B}\right)\right)\frac{\partial e_u^B}{\partial e_p^B}, \tag{11}$$

where

$$B = \frac{y(\ell u, hp) - y(\ell u, \ell p)}{2}.$$

While the LHS strictly decreases in π, the effect on the RHS is ambiguous. A sufficient condition for condition (11) to tighten in π is $A + 2B > 1$ and $2B + A > 1$, that is $A + B = (y(hu, hp) - y(\ell u, \ell p))/2$ sufficiently close to 1.

Since $S^B - S^L = 0$ at $\pi = 1$ this suffices to ensure that $S^B - S^L$ is decreasing in π at each zero for $\pi \in [1/2, 1)$ if $A + B > e_u^L = (y(hu, hu) - y(\ell u, \ell u))/2$ is sufficiently great. This in turn implies that $S^B - S^L$ has a unique zero at $\pi = 1$ on $\pi \in [1/2, 1]$, and is strictly positive on $[1/2, 1)$. A similar argument holds in case $e_p^B < e_u^B$ with $A' = y(hu, \ell p) - y(\ell u, \ell p)$ and $B' = y(hu, hp) - y(hu, \ell p)$, where $A' + B' = y(hu, hp) - y(\ell u, \ell p)$.

In case $\pi < 1/2$ the surplus difference is now

$$\begin{aligned} S^B - S^L =& \pi \frac{(e_p^B)^2 - (e_p^L)^2}{2} + (1-\pi) \frac{(e_u^B)^2 - (e_u^L)^2}{2} - (1-\pi) e_u^B + (1 - 2\pi) e_u^L \\ & + \pi \left(\frac{y(hu, hp) - y(\ell u, \ell p)}{2} + y(\ell u, \ell p) - \frac{y(\ell p, \ell p) + y(\ell u, \ell u)}{2} \right). \end{aligned}$$

Hence, again a sufficient condition for a background based policy to dominate laissez faire is $\pi {e_p^B}^2 + (1-\pi) {e_u^B}^2 > \pi {e_p^L}^2 + (1-\pi) {e_u^L}^2$. The difference $S^B - S^L = 0$ for $\pi = 0$. Its derivative with respect to π is

$$\begin{aligned} \frac{\partial (S^B - S^L)}{\partial \pi} =& - \frac{(e_p^L)^2 - (e_p^B)^2}{2} - \frac{(e_u^B)^2 - (e_u^L)^2}{2} - (1-\pi)(1 - e_u^B) \frac{\partial e_u^B}{\partial \pi} + \pi e_p^B \frac{\partial e_p^B}{\partial \pi} \\ & + \left(\frac{y(hu, hp) + y(\ell u, \ell p)}{2} + e_u^B - 2 e_u^L - \frac{y(\ell p, \ell p) + y(\ell u, \ell u)}{2} \right). \end{aligned}$$

Whenever $S^L = S^B$ the derivative is

$$\frac{\partial (S^B - S^L)}{\partial \pi} = \frac{1}{\pi} (e_u^B - e_u^L) \left(1 - \frac{e_u^L + e_u^B}{2} \right) + \left(\pi e_p^B \frac{\partial e_p^B}{\partial e_u^B} - (1-\pi)(1 - e_u^B) \right) \frac{\partial e_u^B}{\partial \pi}.$$

This is zero for $\pi = 0$, and positive whenever

$$\left(1 - \frac{e_u^L + e_u^B}{2} \right) > \frac{(1-\pi)(1 - e_u^B) - \pi e_p^B \frac{\partial e_p^B}{\partial e_u^B}}{(1-\pi)(1 - e_u^B) - \pi B \left(\frac{1 - e_p^B}{1 - e_u^B} - \frac{\partial e_p^B}{\partial e_u^B} \right)} (1 - e_u^B).$$

Using the definition of e_u^B for $\pi < 1/2$ from (4) this is equivalent to

$$\frac{x_u - e_u^L}{2} \geq \left(1 + \frac{e_u^B - e_u^L}{2(1 - e_u^B)} \right) \left(x_u - A - B \frac{\partial e_p^B}{\partial e_u^B} \right) - e_p^B \frac{\partial e_p^B}{\partial e_u^B}.$$

Note that the condition's slackness decreases in π. For $\pi = 1/2$ with $e_u^B = x_u = e_p^B = A + B$ the above condition holds true when setting $A + B = 1 - \epsilon$ for $\epsilon > 0$ sufficiently small. Similar to above this is sufficient to guarantee that $S^B - S^L$ increases in π at each zero, and therefore $S^B > S^L$ for $\pi \in (0, 1/2]$ if $A + B > e_u^L = (y(hu, hu) - y(\ell u, \ell u))/2$ is sufficiently great. Again, an analogous argument holds in case $e_p^B < e_u^B$. $\square$

Derivatives of e_p^B and e_u^B:

Assuming that $e_u \leq e_p$ and solving (4) for e_u^B yields

$$e_u^B = \frac{1}{2}(1+A) - \frac{1}{2}\sqrt{(1-A)^2 - 4(1-e_p^B)B},$$

where only the positive root is consistent with $e_u^B \leq e_p^B$ if $e_p < A+B$, and only the negative when $A+B \leq e_p^B \leq 1$. Hence,

$$\frac{\partial e_u^B}{\partial e_p^B} = \frac{-B}{|1+A-2e_u^B|},$$

with $\partial e_u^B/\partial e_p^B < 0$ for $e_p^B > A+B$ and $\partial e_u^B/\partial e_p^B > 0$ for $e_p^B < A+B$. Differentiating (3) with respect to π yields

$$\frac{\partial e_p^B}{\partial \pi} = \frac{e_p^L - x_p}{\pi^2} - \frac{1-\pi}{\pi}\frac{A}{e_p^B}\left(-\frac{\partial e_u^B}{\partial e_p^B} + \frac{e_u^B}{e_p^B}\right)\frac{\partial e_p^B}{\partial \pi}.$$

Hence, $\partial e_p^B/\partial\pi > 0$ whenever $e_p^L > x_p$ and $e_p^B \geq A+B$. Suppose $e_p^L > A+B$, then $\partial e_p^B/\partial\pi > 0$ at $\pi = 1/2$. But then $A+B < e_p^B < e_p^L$ for $\pi \in (1/2, 1)$. Supposing the contrary, $e_p^L < A+B$, implies $\partial e_p^B/\partial\pi < 0$ at $\pi = 1/2$ and therefore $A+B > e_p^B > e_p^L$ for $\pi \in (1/2, 1)$. Since $x_p \leq (y(hu, hp) - y(\ell u, \ell p))/2$ also $e_p \leq (y(hu, hp) - y(\ell u, \ell p))/2$ for $\pi < 1/2$.

Proof of Proposition 7

Under a background based policy p agents solve

$$\max_e \begin{cases} ew - \frac{e^2}{2} & \text{if } \pi \leq 1/2 \\ e\left(\frac{1-\pi}{\pi}w + \frac{2\pi-1}{\pi}W\right) - \frac{e^2}{2} & \text{if } \pi > 1/2. \end{cases}$$

Interior solutions satisfy $e^B = w$ if $\pi \leq 1/2$, and $e^B = W - (1-\pi)(W-w)/\pi$ if $\pi > 1/2$. That is, $e^{LF} > e^B \geq w > e^A$ for $\pi \in (0, 1)$.

Derivation of the Cutoff $W_1(w, \pi)$

For $\pi < 1/2$ a background based dominates an achievement based policy. While both policies induce exactly the same matching – each P is matched with a U and there is the same measure of (h, ℓ) firms for a given e – since $e^B > e^A$, there are more integrated firms and, as $2w > W$, surplus is higher. Therefore $W_1(w, \pi) = w$ as claimed.

If $\pi > 1/2$ screening by background loses its effectiveness as a measure $2\pi - 1$ of P agents segregate in education outcome, unlike under an achievement based policy. Total surplus under a background based policy is

$$S^B = e^B \left((2\pi - 1)W + (1-\pi)2w - \pi \frac{e^B}{2} \right). \tag{12}$$

$S^B > S^A$ if and only if

$$\pi(e^B - e^A)\left(2w - \frac{1}{2}(e^B + e^A)\right) > (2\pi - 1)e^B(2w - W). \tag{13}$$

The LHS captures the gain through better incentive provision by a background based policy, while the RHS gives the benefit from rematching by an achievement based policy. Since (13) strictly relaxes as W increases,

$$S^B > S^A \Leftrightarrow W > W_1(w, \pi).$$

Straightforward calculation shows that $W_1(w, \pi)$ increases in π for $\pi \in [1/2, 1]$. $W_1(w, 1) = W_0(w, 1)$, as for $\pi = 1$ a background based policy implies the laissez-faire outcome. Therefore $W_0(w, \pi) > W_1(w, \pi)$ for $\pi < 1$ and the difference decreases in π.

Finally, we show that $W \geq \sqrt{3}w$ implies $S^B > S^A$. Since e^B increases and e^A decreases in π, both output and incentive effect in condition (13) move in the same direction. Since W_1 increases in π, $W_1(w, \pi)$ may not be monotone in w. Solving the quadratic expression $S^B > S^A$ for e^A yields

$$e^A < 2w - \sqrt{4w^2 - \frac{2}{\pi} S^B}. \tag{14}$$

$2S^B > 3\pi w^2$ gives a sufficient condition:

$$\begin{aligned} & 2((2\pi - 1)W + (1-\pi)w)((2\pi - 1)W + (1-\pi)3w)/2 > 3\pi^2 w^2 \\ \Leftrightarrow \quad & (2\pi - 1)W^2 + 4(1-\pi)Ww > 3w^2. \end{aligned}$$

Solving this quadratic expression in W yields the condition

$$W > \left(\frac{\sqrt{3(2\pi - 1) + 4(1-\pi)^2}}{2\pi - 1} - 2\frac{1-\pi}{2\pi - 1} \right) w.$$

Since $(\sqrt{3(2\pi - 1) + 4(1-\pi)^2} - 2(1-\pi))/(2\pi - 1) < \sqrt{3}$ a sufficient condition for $S^B > S^A$ is $W \geq \sqrt{3}w$, that is $W_1(w, \pi) < \sqrt{3}w$.

Derivation of the Cutoff $W_2(w,\pi)$

Compare now a background based policy to laissez-faire when $\pi \leq 1/2$. Under laissez-faire there are $\pi e^{LF}/2$ firms of type (h,h) and total surplus is $S^{LF} = \pi W^2/2$. With a background based policy there are measure πe^B of (h,ℓ) firms and total surplus is $S^B = \pi 3w^2/2$. Therefore, when $\pi \leq 1/2$, $S^{LF} > S^B$ if and only if $W > \sqrt{3}w$; hence $W_2(w,\pi) = \sqrt{3}w$ as claimed.

Consider now the case $\pi > 1/2$. In this case, $S^{LF} > S^{BB}$ if and only if

$$(e^{LF}-e^B)\left[(1-\pi)2w+(2\pi-1)W-\frac{\pi}{2}\left(e^{LF}+e^B\right)\right] > (1-\pi)e^{LF}(2w-W). \tag{15}$$

Manipulating condition (15) and solving for W yields

$$W > \frac{4\pi-2+\sqrt{1-4\pi+7\pi^2}}{3\pi-1}w := W_2(w,\pi).$$

Clearly, $W_2(w,\pi)$ increases in w and simple calculation reveals that W_2 increases also in π. Note that $W_2(w,\pi) \to 2w$ as $\pi \to 1$. Bounds on W_2 for $\pi \in [1/2, 1]$ are given by

$$\sqrt{3}w = W_2(w,1/2) \leq W_2(w,\pi) \leq W_2(w,1) = 2w.$$

$W_2(w,\pi) \geq \sqrt{3}w$ implies that $W_2(w,\pi) > W_0(w,\pi)$ (see Corollary 2).

Supplementary Material

Some Transferability

To see that our results are robust to admitting some transferability in the labor market assume first that surplus is shared equally in firms, but agents have quasilinear utility and can make side payments up to b. This can be thought of cash endowments in the absence of credit markets. Integration under Assumption DD is possible if

$$w+t \geq W \text{ and } w-t \geq 0,$$

where $t \in [-b, b]$ is a side payment from a ℓ agent to a h agent. Hence, if $b < W - w$ utility is sufficiently transferable to ensure that the labor segregates.

Another cause for nontransferable utility is moral hazard within a firm. To illustrate this suppose that output is stochastic and occurs only with probability $\rho(x, x')$, which depends on effort x and x' chosen by the firm members. Suppose that effort x induces a utility cost of $x^2/2$ and that

$$\rho(x, x') = (x + x')^\delta,$$

where $\delta \in [0, 2)$ is a parameter indicating the responsiveness of the success probability to effort. Let output depend on firm members' achievements through $y(a, a')$ and assume that expected surplus in firm is

$$(x + x')^\delta y(a, a')^{\frac{2-\delta}{2}} - (x^2 + (x')^2)/2.$$

The exponent $(2-\delta)/2$ serves to compensate scale effects from larger projects $y(a, a')$ on effort choice. This ensures that desirability of integration is governed by the properties of $y(.)$ independent of δ. Both partners in the match can contract on the sharing of surplus ex ante, at the matching stage. Denote the individual shares by s and s' with $s + s' = 1$. Given a sharing rule s optimal individual effort choice satisfies for $\alpha > 0$

$$x = s\delta(x + x')^{\delta-1} y(a, a')^{(2-\delta)/2}, \tag{16}$$

and analogously for x'. This yields $(1 - s)x = sx'$ and the solutions

$$x(s) = s\delta^{\frac{1}{2-\delta}} y(a, a')^{1/2} \text{ and } x'(s) = (1 - s)\delta^{\frac{1}{2-\delta}} y(a, a')^{1/2}.$$

The Pareto frontier is then given by the solution to

$$\max_{s\in[0,1]} \left(\delta^{\frac{\delta}{2-\delta}} - \frac{s\delta^{\frac{2}{2-\delta}}}{2}\right) s y(a, a') \text{ s.t. } \left(\delta^{\frac{\delta}{2-\delta}} - \frac{(1-s)\delta^{\frac{2}{2-\delta}}}{2}\right)(1-s)y(a, a') \geq u_b.$$

Either the constraint binds, and

$$1 - s = \frac{1}{\delta} - \frac{\sqrt{\delta^{2\frac{\delta-1}{2-\delta}} - \frac{2u_b}{y(a,a')}}}{\delta^{\frac{1}{2-\delta}}}. \tag{17}$$

This expression defines $s(u_b)$. Otherwise maximizing u_a does not constrain u_b, so that $s = \frac{1}{\delta}$. Note that this is only possible for $\delta > 1$. Using the expression (17) the Pareto frontier can be written as

$$u_a(u_b) = s(u_b)\delta^{\frac{2}{2-\delta}} \left(\frac{1}{\delta} - \frac{s(u_b)}{2}\right) y(a, a'), \tag{18}$$

whenever the constraint binds, i.e., the following condition holds:

$$\frac{3 - \delta}{2}(\delta - 1)\delta^{2\frac{\delta-1}{2-\delta}} y(a, a') \leq u_b, \tag{19}$$

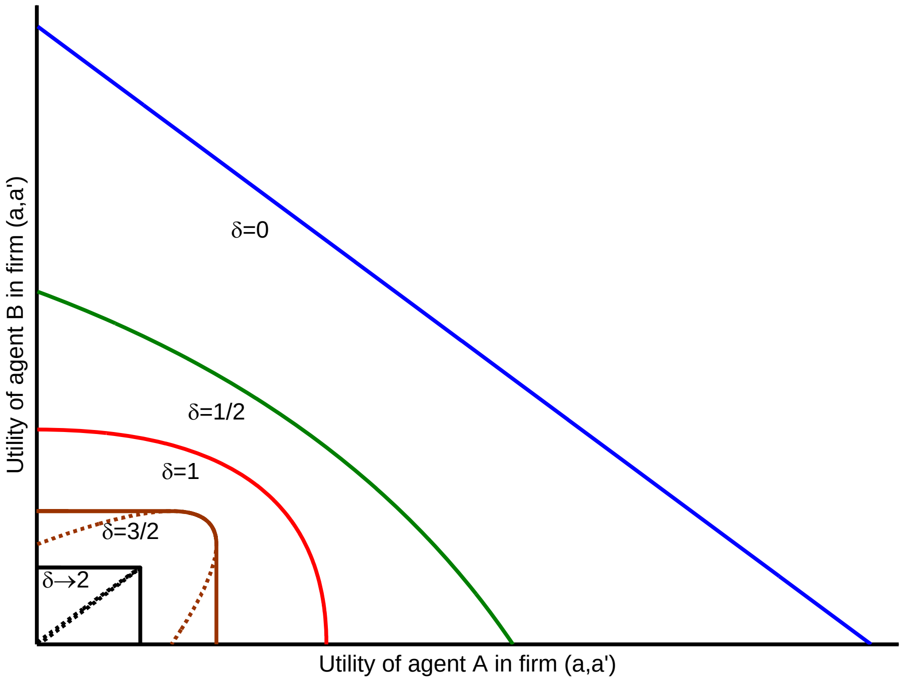


Figure 2: Partners' utilities corresponding to choices of s given δ.

and otherwise

$$u_a(u_b) = \delta^{2\frac{\delta-1}{2-\delta}} \frac{y(a,a')}{2}. \tag{20}$$

Note that $u_a(u_b)$ defined by (18) is concave in u_b.

Figure 2 shows the Pareto frontiers (solid lines) in partnership problems associated to different values of δ. In case $\delta \leq 1$ agents' utilities corresponding to effort choices $x(s)$ and $x'(s)$ for all $s \in [0,1]$ determine the Pareto frontier given by (18). Clearly, for $\delta = 0$ it is optimal to set $x = 0 = x'$, so that in this case utility is perfectly transferable (the Pareto frontier is a straight line with slope -1), since $\rho(x,x') = 1$ independently of effort choice and thus of s. As δ increases, so does the curvature of the Pareto frontier, decreasing transferability by making compensations more costly in terms of total surplus. For $\delta > 1$ the constraint (19) may not bind, so that the locus of agents' utilities that can be reached by choice of $s \in [0,1]$ may be bend backwards, as depicted by the dashed lines. In that case the points on the Pareto frontier are given by (20).

Segregation payoffs in a firm (a,a) share the surplus equally and are

$$\underline{u}(a) = \delta^{\frac{2}{2-\delta}} \left(\frac{1}{2\delta} - \frac{1}{8} \right) y(a,a).$$

Note that equal incentives (and effort provision) is efficient (see Ray et el., 2007) so that desirability of integration depends indeed on the properties of $y(.)$. Integration will maximize total surplus if and only if $2y(a,a') > y(a,a) + y(a',a')$, which is implied by Assumption DD.

A necessary condition for integration is that wages in a firm (a,a') can be chosen to exceed segregation payoffs of both firm members. Letting $a > a'$, this is expressed in the following condition:

$$u_a(\underline{u}(a')) \geq \underline{u}(a). \tag{21}$$

The following proposition states that there always is an effort provision technology characterized by parameter δ such that integration is not sustainable in equilibrium, i.e., (21) fails.

Proposition 8 *For any* $0 \leq y(a',a') < y(a,a') < y(a,a)$ *there exists a* $\delta^* < 2$ *such that condition (21) fails for all* $\delta \in (\delta^*, 2)$*. If* $y(a',a') = 0$ *and* $y(a,a)/y(a,a') > 4/3$*,* $\delta^* = 1$*.*

Proof: If condition (19) holds, (21) can be rewritten using (18) to yield

$$s(\underline{u}(a'))\delta^{\frac{2}{2-\delta}}\left(\frac{1}{\delta} - \frac{s(\underline{u}(a'))}{2}\right) y(a,a') \geq \delta^{\frac{2}{2-\delta}}\left(\frac{1}{2\delta} - \frac{1}{8}\right) y(a,a), \tag{22}$$

and otherwise

$$\delta^{2\frac{\delta-1}{2-\delta}}\frac{y(a,a')}{2} \geq \delta^{\frac{2}{2-\delta}}\left(\frac{1}{2\delta} - \frac{1}{8}\right) y(a,a).$$

This can be rewritten as

$$\bar{y} \leq \frac{1}{\delta - \delta^2/4}, \tag{23}$$

where $\bar{y} = y(a,a)/y(a,a') > 1$. Hence, a sufficient condition for (23) to fail is $\delta - \delta^2/4 > 1/\bar{y}$. By continuity there exists $\delta_0 < 2$ such that this is the case for all $\delta \in (\delta_0, 2)$. Indeed $\bar{y} > 4/3$ ensures that this is the case whenever (19) fails. (19) can be rewritten in case $u_b = \underline{u}(a')$ as

$$\frac{(3-\delta)(\delta-1)}{\delta - \delta^2/4} \leq \underline{y},$$

where $\underline{y} = y(a',a')/y(a,a') < 1$. As above, by continuity there exists $\delta_1 < 2$ such that the above condition fails for all $\delta \in (\delta_1, 2)$. Hence, for any technology $\bar{y} > 1 > \underline{y}$ there is a $\delta^* = \max\{\delta_0; \delta_1\}$ sufficiently close to 2 such that both (19) and (23) fail for all $\delta \in (\delta^*, 2)$. For $y(a',a') = 0$ we have $\delta_1 = 1$ and $\delta^* = \delta_0$. □

Turning back to (22) for the sake of completeness and solving for $s(.)$ yields

$$s(\underline{u}(a')) \geq \frac{1}{\delta}\left(1 - \sqrt{1 - \bar{y}\left(\delta - \frac{\delta^2}{4}\right)}\right),$$

By (17)

$$s(\underline{u}(a')) = \frac{\delta - 1}{\delta} + \frac{\sqrt{\delta^{2\frac{\delta-1}{2-\delta}} - \frac{2\underline{u}(a')}{y(a,a')}}}{\delta^{\frac{1}{2-\delta}}}.$$

Condition (22) takes care of nonnegativity of the term under the root. Combining the last two expressions yields a necessary condition for integration

$$\sqrt{1 - \underline{y}\left(\delta - \frac{\delta^2}{4}\right)} + \sqrt{1 - \bar{y}\left(\delta - \frac{\delta^2}{4}\right)} \geq 2 - \delta.$$

Since $\underline{y} \geq 0$, the above condition fails whenever $\delta < 1$ and

$$\bar{y} > \frac{2 - \delta}{1 - \delta/4}.$$

Kremer-Maskin Foundation

Note that a Kremer-Maskin task assignment model satisfies most of these conditions by design. The Kremer-Maskin production function is defined as

$$y(s_i, s_j) = \max\{s_i^{\theta} s_j^{1-\theta}; s_i^{1-\theta} s_j^{\theta}\},$$

where $1/2 < \theta < 1$ and $s_i, s_j \in \mathbb{R}$ are skill levels of firm members i and j. Clearly, the function y is monotone in its arguments, and

$$2\max\{s_i^{\theta} s_j^{1-\theta}; s_i^{1-\theta} s_j^{\theta}\} > s_i + s_j,$$

whenever $\theta > 1/2$ so that property (DD) is satisfied. Suppose $\ell u, hu, \ell p, hp$ are real valued and normalize $hp - \ell u < 2$. Note that $y(.)$ has property (DC) if

$$hu - \ell u < \ell u^{1-\theta}(hp^{\theta} - \ell p^{\theta}) + (hu^{1-\theta} - \ell u^{1-\theta})\ell p^{\theta} \text{ and } hu - \ell u < 1.$$

This holds for instance if hu and ℓu are sufficiently close. Clearly, $y(.)$ satisfies the first part of property (M). The second part holds if

$$\frac{\ell u^{1-\theta}}{hp^{\theta}} > \frac{\ell p^{1-\theta} - hu^{1-\theta}}{\ell p^{\theta} - hu^{\theta}}.$$

That is, the support of the skill distribution has to be sufficiently tight.

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