

# Smithian Growth Through Creative Organization\*

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## Abstract

We consider a model in which appropriate organization fosters innovation, but because of contractibility problems, this benefit cannot be internalized. The organizational design element we focus on is the division of labor, which as Adam Smith argued, facilitates invention by observers of the production process. However, entrepreneurs choose its level only to facilitate monitoring their workers. Whether there is innovation depends on the interaction of the markets for labor and for inventions. A high level of specialization is chosen when the wage share is low. But low wage shares arise only when there are few entrepreneurs, which limits the market for innovations and therefore discourages inventive activity. When there are many entrepreneurs, the innovation market is large, but the rate of invention is low because there is little specialization. Rapid technological progress therefore requires a balance between these opposing effects, which occurs with a moderate relative scarcity of entrepreneurs and workers. In a dynamic version of the model in which a credit constraint limits entry into entrepreneurship, this relative scarcity depends on the wealth distribution, which evolves endogenously. There is an inverted-U relation between growth rates driven by innovation and the level of inequality. Institutional improvements have ambiguous effects on growth. In light of the model, we offer a reassessment of the mechanism by which organizational innovations such as the factory may have spawned the industrial revolution.

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# 1 Introduction

Ideas drive technological change. And as with the production of other goods, the production of ideas depends on the organizational environment. Understanding technological change therefore requires an understanding of the relationship between organization and invention. What kinds of organization best foster the generation of new ideas? What are the broader economic conditions in which such creative organization will emerge?

Recent efforts to answer these questions have focused, naturally enough, on the case in which invention happens to be an objective of the firm, as in the R&D laboratory or a new-product joint venture.<sup>1</sup> In this situation, the organization is designed to maximize the value of the inventions produced at least cost, taking into account constraints both on incentives and technology – in particular the “technology” of human cognition. Environments that are generally favorable for investment will be conducive to innovation in particular.

A wider look at the economic development process suggests, however, that much of historical technological change has occurred without the involvement of such firms. Organized R&D was rare before the twentieth century; today it is confined to only a relatively small fraction of firms worldwide and is responsible for only a fraction of innovation. Nevertheless, even when it has other objectives, the firm remains the main arena of innovation: technological progress can be an *unintended* consequence of organizational design. This paper presents a first attempt at modeling this aspect of the determinants of economic growth.

The industrial revolution provides the natural backdrop for examining the issues. A distinguishing feature of the period was the rise of the factory system, in which production was carried out by workers gathered under one roof, with strict supervision and discipline, and most important perhaps, a division of labor. And though a vast literature reveals wide assent on the importance of the factory system, economic historians display little consensus — no doubt fueled by the wide cross-country variation in the manner and timing of its adoption — on just what role it actually played in fostering the rapid technological advances and economic growth that also char-

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<sup>1</sup>See, e.g., Aghion and Tirole (1994), Garicano and Rossi-Hansberg (2007) as well as antecedents in the industrial organization literature, e.g. Kamien-Schwartz,(1976) and Loury (1979). The perspective that invention should be understood as endogenous to economic forces has been forcefully advanced by some of the scholars of “new growth theory,” e.g., Romer (1986, 1990) and Aghion-Howitt (1992).

acterized the era. Some commentators, like Landes (1969), seem to argue that the factory was epiphenomenal, merely an optimal organizational response to exogenous technological change. Others, like Cohen (1981), Millward (1981), and North (1981), suggest that it was the enhanced efficiency of the factory system itself relative to earlier forms of organization that generated greater surpluses, though it is difficult to see how this by itself could plausibly translate into increased rates of innovation and growth.

A third view, attributable to Adam Smith and echoed by later writers on the industrial revolution such as Charles Babbage and Amsa Walker, affirms a causal role for the factory system. It places the emphasis less on its static benefit of making better use of current inputs to produce current output, than on a *dynamic* one: the adoption of the factory, and in particular the fine division of labor into elementary tasks, engenders a “cognitive externality” by providing a superior environment to inspire invention and refinement of productive techniques. Primarily these inspirations accrue to persons, such as the occasional workman or an outside observer, other than the factory’s owner. By focusing an individual’s attention, it makes it easier for him to improve on old techniques. Alternatively, and complementarily, by providing a “model” in human form of elementary tasks, it facilitates the development of machines that can better perform those tasks.<sup>2</sup> This effect seems especially pertinent to understanding the development of micro-inventions, the sustained flow which was a crucial element of the industrial revolution (Rosenberg 1982, Mokyr 1990).

Now, there is little evidence that anyone during the industrial revolution ever built a factory because he expected it to help him innovate. Given the nonrival nature of ideas and the difficulties in excluding them for long, this should not be surprising. More difficult still, but essential for appropriating a return to establishing a creative organization, would be proving the source of inspiration for an idea that could be widely applied. Thus, the creative role of the division of labor could only be harnessed via some other economic mechanism that would have induced the widespread adoption of the factory and the concomitant surge of technical progress.

Fortunately, the division of labor had other benefits, as Smith himself enumerated. Among them was the enhanced ability to monitor labor: a worker assigned to only

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<sup>2</sup>From the *Wealth of Nations*: “I shall only observe, therefore, that the invention of all those machines by which labor is so much facilitated and abridged seems to have been originally owing to the division of labor. Men are much more likely to discover easier and readier methods of attaining any object when the whole attention of their minds is directed towards that single object than when it is dissipated among great variety of things.”

a small number of tasks will be less able to disguise shirking as downtime between tasks, or to find opportunities to embezzle either inputs or outputs undetected.<sup>3</sup> This idea has been rekindled by the burgeoning literature on multi-task principal-agent problems following Holmström and Milgrom (1991) and Baker (1992), which highlights the gains from better incentives that come from reducing the number of tasks assigned to a single worker. Thus, an entrepreneur could benefit privately from a fine division of labor, even if he had no interest in its social benefit in terms of eased invention.

Of course dividing labor does not come for free. There are lost economies of individual scope from minute task division; costs of communication among specialized workforces; time spent coordinating tasks among disparate individuals; and resources that have to be marshaled for assembling the components of the final good produced by each worker (see for instance, Becker and Murphy 1992, Radner and Van Zandt 1992, Bolton and Dewatripont 1994, Garicano 2000, Dessein and Santos 2006). Entrepreneurs would have to trade off the monitoring benefit of labor division against these coordination costs.

A basic implication of incentive theory is that monitoring benefits vary with the wage level. In particular, entrepreneurs would be induced to choose a high level of labor division only when wages are low, requiring more monitoring (this relationship between wage and monitoring levels has also appeared in the efficiency wage literature, e.g., Acemoglu and Newman 2002). Thus, conditions in the labor market will determine an entrepreneur's choice of labor division, and via the cognitive externality, the level of technological progress.

Before delving into the mechanism by which the interaction of the monitoring and invention benefits of labor division can provide an organizational theory of growth, we note that we are abstracting from the third, well-known benefit of labor division, which is the first one Smith mentions in *Wealth of Nations*: “improved dexterity,” or direct productivity gain. Because it is more “technological” than “organizational,” it provides less of a clear tradeoff that can account for the wide cross-country variation in degree of labor division and the use of the factory system. Thus when we refer to

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<sup>3</sup>Also from the *Wealth of Nations*: “A man commonly saunters a little when turning his hand from one sort of employment to another...The habit of sauntering and of indolent careless application, which is naturally, or rather necessarily, acquired by every country workman who is obliged to change his work and his tools every half hour and to apply his hand in twenty different ways almost every day of his life, renders him almost always slothful and lazy, and incapable of any vigorous application even on the most pressing occasions.”

“division of labor” or “specialization,” we mean beyond whatever level at which the marginal improved dexterity equals the marginal cost of labor division. See footnote 10 for further discussion.

The setting for our investigation is a standard occupational choice model (e.g., Banerjee and Newman 1993). Individuals are either workers, who supply imperfectly observable effort to firms, or entrepreneurs, who hire workers and choose the degree of labor division within their firms. The market wage will mediate both the occupational and organizational choices, since it affects the relative attraction of the occupations and the returns to monitoring. The relative scarcity of entrepreneurs and workers determines the wage and through it, the organizational design of firms.

A special class of individuals whom we dub “nerds” are the ones who tinker with the old technology and find ways to improve it.<sup>4</sup> Like everyone else, they respond to incentives, choosing how much to invest in inventive effort partly on the basis of how much they expect to earn selling any inventions they may produce. But the ease of inventing is determined also by the degree of labor division in the economy’s firms: fine division makes it easy to invent – e.g., to replace a human performing a simple repetitive task by a machine that can do the same much faster. With a coarse division of labor, perceiving which aspects of a job are subject to improvement or mechanical replacement is much more difficult.<sup>5</sup>

As in earlier occupational choice models, there are different types of equilibria, uniquely determined by the relative scarcity of entrepreneurs and workers, and characterized in part by the nature of the predominant organizational forms. But here, equilibria are also characterized by the accompanying rate of technological innovation.

There is an “artisanal” equilibrium in which workers are responsible for a large number of tasks (i.e., a low degree of labor division) and wage shares are high. There is also a “factory” equilibrium with finely divided labor and lower wage shares. The artisanal equilibrium is the statically more efficient one, since fewer resources are lost to monitoring via the costly division of labor. But it may be dynamically inefficient in the sense that the innovation rate is low, owing to the difficulty of inventing under

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<sup>4</sup>Making the inventors a special population is mainly for analytical simplicity – it would not change things much to assume they are drawn from the population at large. See Khan and Sokoloff (1990) for evidence on the social background of inventors during the industrial revolution.

<sup>5</sup>An alternate and complementary interpretation is suggested by Aoki (1986): with a coarse division of labor, much of the information about the various tasks will remain tacit; with a fine division of labor, coordination may entail formal codification of this knowledge. Once documented, however, the knowledge becomes more accessible to inventors, which facilitates technological improvement.

a coarse labor division.

The factory equilibrium, though statically wasteful, has the potential to generate higher rates of innovation than the artisanal one. Since it only exists when there are few entrepreneurs, this might seem to imply that entrepreneurship actually impedes innovation, the artisanal equilibrium being a case of too much of a good thing. But there is a countervailing effect. Since nerds will be able to sell their inventions to the entrepreneurs on an innovation market, the investment decision for an aspiring inventor depends on the extent of this market: the larger it is, the more revenue is available.

The scarcity of entrepreneurs therefore affects the innovation market as well as the labor market through a *market size effect*. When there are relatively many workers, the number of buyers of inventions is small, and though it may be easier to invent, the revenue generated will be too small to justify the effort: the innovation market shuts down.<sup>6</sup> At the other extreme, despite a large demand for innovations that comes when there are many entrepreneurs, innovation will be undermined by the difficulty of inventing under the artisanal mode of firm organization. Moderate ratios of entrepreneurs to workers however, keep wage shares low so that specialization is high and ideas arrive easily, and at the same time provides enough of a market for them to induce people to invent. In short the model predicts an inverted U-shaped relation between the fraction of the population who are entrepreneurs and the rate of technological progress.

Turning to long-run dynamics, the model can generate steady-state endogenous growth. We endogenize entry into the occupations by supposing there is a credit market imperfection that inhibits those with less than a threshold level of wealth from becoming entrepreneurs. Thus, the proportion of entrepreneurs will be identified with the fraction of “rich” agents, which becomes the state variable for the economy. We demonstrate the existence and local stability of steady states in which the static relation between the entrepreneur-worker (now the rich-poor) ratio and the rate of innovation and growth is maintained in the long run. An economy initially with many poor will tend to collapse to a pure subsistence equilibrium. One with many rich will make technological change slow or nonexistent, though it may appear

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<sup>6</sup>Something like this size effect can be gleaned by comparing industries in eighteenth century Britain. Watchmaking had a fine division of labor going back at least a century earlier, but it served only a small (luxury) market, and thus never experienced the high levels of innovation that affected other industries such as cotton and steel (Mathias 1983).

statically affluent, with a high wage share and few resources lost to coordinating divided labor. Only economies that initially have moderate inequality will be able to sustain a high rate of steady-state innovation and growth.<sup>7</sup> We discuss how the basins of attraction of these steady states depend on “institutional” factors such as the quality of financial markets, the complexity of technology, and most important, organizational innovations such as the demise of the putting-out system and rise of the factory.

These results suggest a possible explanation for the venerable economic historians’ conundrum of why Britain among European nations was first to industrialize. Compared with some of its continental counterparts (notably France), in the late eighteenth century both had similar levels of technology, some form of patent system, free labor markets, and (as in our model) only rudimentary and imperfect credit markets. Yet France remained a nation of family farms and small enterprise for several decades, while Britain rapidly became a nation of factories and the seat of the Industrial Revolution (Deane 1965, Shapiro 1967, O’Brien and Keyder 1978, Crafts 1985, Crouzet 1990, Mokyr 1990). One difference was the distribution of wealth, which was rather more unequal in England (Clapham 1936, Grantham 1975, Soltow 1980). Other slow-to-industrialize countries, such as (northern) Italy, had greater inequality than Britain in the same period.

In addition, the analysis offers a novel perspective on precisely how the organizational innovation that was the factory contributed to the Industrial Revolution. Centralizing production under one roof (the “manufactory”) rather than decentralizing it in worker’s cottages (the “putting-out system”) would have reduced the cost of dividing labor (e.g. lower costs of transporting partly finished products from one worker to the next), even though logically, one might have had a very fine division of labor under putting-out as well. It would also have increased the monitoring benefit, since enforcement of rules against straying from one’s work station obviously would have been cheaper to enforce in the manufactory than under putting out. Thus, the adoption of the manufactory would have led to a finer division of labor, facilitating invention as we have suggested, and ultimately giving us the Industrial Revolution. Of course, the manufactory did not accomplish this in isolation – certain accessory institutional and distributional conditions were satisfied as well, in Britain especially.

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<sup>7</sup>As far as we know, such an “inverted U” relationship between inequality and innovation is new to the literature. In our case, inequality not only governs the innovation rate, but also is influenced by it, since it is determined in part by the incomes accruing to the inventors.

We shall have more to say on this in the Conclusion.

## 2 The Basic Model

In this section we consider a “static” model in which the occupation of each agent is exogenous. In the next section we extend the model dynamically: agents make an occupational choice that is partially constrained by their wealth, which evolves endogenously.

### 2.1 Agents and Timing

Economic activity takes place at two dates, 1 and 2. At each date, there is a measure  $1-\eta$  or “normal” individuals who are economically active; of these,  $r$  are *entrepreneurs* and  $1-r$  are *workers*. In addition, there are  $\eta$  *nerds* who are active at both dates (they are “young” at date 1 and “old” at date 2). All agents are risk neutral and are endowed with a unit of effort: normal agents use it to produce the economy’s single consumption good; nerds use it to produce inventions.

Entrepreneurs can each hire up to  $n$  workers, or can operate on their own in “artisan” firms. Young nerds observe the production process carried out by the normals active at date 1. Those who succeed in finding an idea for improving technology may enter the innovation market to sell their inventions to the entrepreneurs who are active at date 2.

### 2.2 Production

#### 2.2.1 Technology

Production of the consumption good involves a unit measure of “jobs” indexed by  $j \in [0, 1]$ . The labor productivity for job  $j$  is  $a(j)$  and output is  $\exp(\int_0^1 \log[a(j)l(j)]dj)$ , where  $l(j)$  is the labor allocated to job  $j$ . Given this technology, labor is uniformly allocated over all jobs independently of  $a(\cdot)$ . Output per unit labor is therefore  $A = \exp(\int_0^1 \log a(j)dj)$ .

A firm’s output is linear in the number of workers hired, up to the scale  $n$ . If the entrepreneur operates on his own without workers, he produces  $\alpha A$ , where  $\frac{n}{2} \geq \alpha \geq 1$ . Thus if an entrepreneur hires any workers at all, he must be making positive profit from each, and therefore will hire  $n$  of them.

We use a “quality ladder” approach for modeling technical progress (Grossman and Helpman 1991). An invention improves the productivity of a single job by the multiplicative factor  $(1 + \gamma)$ . If  $m(j)$  improvements are implemented on job  $j$ , its productivity becomes  $a'(j) = (1 + \gamma)^{m(j)}a(j)$ .<sup>8</sup>

Denote the technology operated by the entrepreneurs at date 1 by  $A = \exp(\int_0^1 \log a(j) dj)$ . This technology is freely available to the date 2 entrepreneurs, though typically they will choose to improve it: if they purchase  $m$  innovations, the new technology is  $A' = A(1 + \gamma)^m$  where  $m = \int_0^1 m(j) dj$ . Since each nerd can have at most one invention, we have  $m \leq \eta < 1$ .

### 2.2.2 Contractibility Assumptions

We make the following assumptions:

1. Worker effort is imperfectly observable; the degree of imperfection depends negatively on the division of labor.
2. Individual output is not contractible.
3. The source of ideas is not attributable, hence entrepreneurs cannot claim ownership of them.
4. Nerd effort is not observable.

As we already discussed in the Introduction, the first assumption provides the basis on which the division of labor benefits entrepreneurs. The second assumption prevents the use of output contingent contracts such as piece-rates.<sup>9</sup> The last two assumptions are the reasons why entrepreneurs do not internalize the effects of labor division on invention: the third one prevents contracting with a nerd on the contingency that he obtained an idea because of the entrepreneur’s organizational choice; combined with the last assumption, it prevents entrepreneurs from establishing “invention factories,” wherein they hire nerds to produce ideas in return for wages.

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<sup>8</sup>Thus one job may be improved an indefinite number of times. This is a simplifying assumption that avoids the computation of the individual returns to inventing when there is a possibility of duplication or diminished returns. Our conclusions would not be altered substantially if it were relaxed, and it gives the best chance for innovation via the Smithian mechanism. It is also convenient for representing steady-state growth without the added complication of providing a theory of how new sectors come to into being during the development process.

<sup>9</sup>This is only for simplicity. Indeed, it is well understood from the multi-tasking literature that piece rates create poor incentives for quality provision. One solution is to separate tasks; thus piece rates and labor division are complementary, in much the same way as input monitoring and labor division. As it happens, available evidence suggests that piece rates were not widely used during the industrial revolution (e.g., Huberman 1996).

### 2.2.3 Division of Labor

Here we describe how the division of labor is modeled and how it facilitates monitoring. The set of jobs can be subdivided into a number of (equal-size) *components*. Denoting their number by  $\sigma$ , each one contains  $1/\sigma$  jobs; hence  $\sigma = 1$  corresponds to the early manufacturing days where artisans were put under the same roof but continued to do all the jobs involved in producing the good, while  $\sigma \geq 2$  may correspond to an assembly line system. Workers are specialized in producing individual components; given the production technology, in order to produce a unit of the good, it is necessary to combine one unit of each of the  $\sigma$  components.

When  $\sigma = 1$ , a worker spreads his unit of labor time uniformly over all the jobs; hence a worker has to spend  $1/\sigma$  of units of labor time to produce one unit of a component consisting of  $1/\sigma$  jobs. Absent coordination problems, it does not make a difference in terms of total output whether each of  $n$  workers does all the jobs (is completely unspecialized), or is  $\sigma$ -specialized, with  $n/\sigma$  workers assigned to each component and producing  $\sigma$  components each: either way, output is  $nA$ .

However, as in Becker and Murphy (1992), we assume that specialization generates coordination problems. For instance, in an assembly line, each worker has to spend time taking the component from the previous worker in line, assembling it with his own component and passing everything to the next component. In many firms producing complex products, seamless integration between components often requires a large number of meetings, reducing time available for production.

When there are  $\sigma$  components, each worker specialized in one component will have to spend time coordinating with  $\sigma - 1$  producers of the other components and the cost in time units is  $c(\sigma - 1)$ , with  $c > 0$ .

Hence, total time available for production is now only  $1 - c(\sigma - 1)$ , so a worker can produce  $\sigma(1 - c(\sigma - 1))$  components. Since there are  $n$  workers per firm,  $n/\sigma$  workers are assigned to each component, and if the entrepreneur's technology is  $A$ , total output is  $n(1 - c(\sigma - 1))A$ .

Compensating for the coordination cost of specialization is its monitoring benefit.<sup>10</sup> In our model, a worker “shirks” not so much by withholding effort but by

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<sup>10</sup>As mentioned earlier, we assume there is no *net* output gain from specialization (unlike e.g., Costinot, 2005); that is, we consider situations where the cost of coordination dominates the productivity gains from specialization. If there are diminishing returns to specialization in this dimension (Smith himself suggests this: see discussion at the end of Section 2.3.3) without a concomitant fall in coordination costs, the net effect of specialization on productivity is first positive and then negative as specialization increases. Since the monitoring benefit increases with specialization, in the first

engaging in a sideline activity – for instance diverting parts to assemble and sell himself – that has a return  $\mu A$ ,  $\mu < 1$ , where  $A$  is the technology available *within the firm*.

It is not possible to distinguish a worker doing job  $j$  for the firm or for himself. A worker assigned to a component consisting of  $1/\sigma$  jobs will spend only  $1/\sigma$  of his time on jobs that are part of that component if he shirks, whereas he spends all his time on the component if he works. Random monitoring will therefore detect shirking with probability  $1 - \frac{1}{\sigma}$ ; hence the higher the level of labor division  $\sigma$ , the more effective is monitoring.

#### 2.2.4 Labor Contracting

On the labor market, entrepreneurs offer contracts  $(\sigma, w)$  consisting of a *degree of specialization*  $\sigma$  and a wage  $w$  normalized to the state of technology  $A$  that is paid only if the worker is not caught shirking.

If the worker is caught shirking, it is optimal to punish him maximally: he loses both his wage  $wA$  and the “booty”  $\mu A$ . Since the shirking worker escapes detection with probability  $1/\sigma$ , shirking yields him a benefit of  $(w + \mu) A/\sigma$  while working yields  $wA$ . It follows that the worker will work when the following incentive compatibility condition is satisfied

$$w \geq \frac{\mu}{\sigma - 1}. \quad (1)$$

Admissible contracts  $(\sigma, w)$  must satisfy (1) as well as a participation constraint.

Observe that higher  $\mu$  implies higher  $\sigma$ , given the wage. Evidence on factory organization in the late eighteenth and early nineteenth centuries is consistent with this prediction. Factories producing easily-embezzled goods (high  $\mu$ ) had higher degrees of labor division than others: watches, for example – valuable goods that could easily be stolen and sold for close to full market value – were produced via a minute division of labor since the early eighteenth century (Mathias 1983, p. 126; Thompson 1963, p. 66.). Similarly luxury-market-oriented goods such as coaches and pianos (low  $\mu$ )

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region it is profit maximizing to increase specialization. We can then interpret  $\sigma = 1$  as a normalization, the maximum degree of specialization for which there are net output gains. If direct gains from specialization were the only appropriable benefit of the division of labor, the question of whether there is economic growth is reduced to the exogenous parametric question of whether specialization gains diminish quickly or slowly enough relative to the innovation gains. Moreover, as we shall note at the end of Section 2.2.4, the monitoring benefit also predicts a positive correlation between productivity and specialization, so the evidence for the productivity benefit must be interpreted with care.

were still produced via traditional techniques as late as the 1830s.<sup>11</sup> In addition to this monitoring effect, smaller components would often tend to have thinner markets, making them less tempting to embezzle (i.e.,  $\mu$  could be a decreasing function of  $\sigma$ , which would reinforce the decreasing relationship between the wage and the degree of division of labor expressed in (1)).

More generally, economic historians have emphasized embezzlement and eliciting worker effort as major concerns in shaping the organization of the first factories and as the chief reasons for the factory system's supplantation of the putting-out system (e.g., Clark 1994, Pollard 1965).<sup>12</sup>

Suppose a firm has technology  $A$ , and let  $u^*$  be the outside option of a worker. The contract that a firm offers solves:

$$\max_{(w,\sigma)} n(1 - c(\sigma - 1) - w)A \quad (2)$$

$$w \geq \frac{\mu}{\sigma - 1} \quad (3)$$

$$wA \geq u^*, \quad (4)$$

where (3) is the worker incentive compatibility constraint, and (4) is the participation constraint.

The incentive constraint binds: if it does not, the entrepreneur can increase her profit by lowering  $\sigma$ . Writing  $\pi(w) \equiv n(1 - \frac{c\mu}{w} - w)$ , the problem reduces to:

$$\begin{aligned} \max_w \quad & A\pi(w) \\ \text{s.t.} \quad & wA \geq u^*. \end{aligned} \quad (5)$$

The unconstrained maximum occurs at  $\underline{w} = \sqrt{c\mu}$ , with concomitant specialization  $\bar{\sigma} = (1 + \sqrt{\frac{\mu}{c}})$  and normalized profit  $\pi(\underline{w}) = n(1 - 2\sqrt{c\mu})$ . Clearly, if the labor market is ever to be active, entrepreneurs must prefer to hire workers (use  $n \geq 2$ )

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<sup>11</sup>See Dodd (1821, pp. 387-408, 432-456) for descriptions piano and coachmaking factories. Pollard (1965, pp. 45, 84-85) offers similar evidence that in shipbuilding or housing (both low- $\mu$ ), production remained organized around the individual craftsman well into the nineteenth century.

<sup>12</sup>Pollard (1965, p.184) discusses work rules that resulted in dismissal for being "found a yard out of his ground," or fines for being "found from the usual place of work, except for necessary purposes, or talking to anyone out of their own Ally [sic]," which would be difficult to implement and enforce without a high level of labor division. Except for products that could be assembled on a small table, a worker with wide responsibility would likely have to wander around the factory and talk to numerous other workers.

rather than work by themselves (in which case their normalized income is  $\alpha$ ) when the wage assumes this minimum value, i.e.,  $\sqrt{c\bar{\mu}} < \frac{1}{2} \frac{n-\alpha}{n}$ . We shall assume that this condition holds for all  $\alpha > n/2$ , that is:

**Assumption 1.**  $\sqrt{c\bar{\mu}} < \frac{1}{4}$ .

The equilibrium wage share is a function of  $u^*$ . For low values, entrepreneurs are not constrained and can choose  $\underline{w}$ . For high values, the participation constraint binds, and entrepreneurs increase the wage beyond  $\underline{w}$  and choose less specialization.

Observe that when the participation constraint binds, if one firm has better technology than another, it will also have a finer division of labor. Since  $wA = u^*$ , raising  $A$  lowers  $w$  and therefore raises  $\sigma$ : a firm with higher  $A$  has more to pilfer (or more to lose when its workers shirk), and this must be offset by more intensive monitoring. Thus, the model would predict a positive correlation between productivity and specialization in a cross section of heterogeneous firms, but higher productivity is the cause, not the consequence, of greater specialization.

### 2.2.5 Invention

Entrepreneurs choose the degree of specialization by considering only the tradeoff between coordination costs and monitoring benefits. What they do not consider is that specialization also affects how easily other agents can find ways to improve the productivity of tasks. These agents are represented by the nerds in our model.

A nerd can generate an idea on how to improve one job. If he *cogitates* during his youth, he observes the state of the art technology  $A$  together with the division of labor  $\sigma$ . He then randomly selects a component for study and arrives at an idea for improvement to one of the tasks in his component with probability  $p(\sigma)$ , where  $p(\cdot)$  is an increasing function. If instead he *vegetates*, he simply generates  $\theta A$  ( $\theta < 1$ ) units of the consumption good for himself.

If an invention is obtained, the nerd becomes active on the innovation market when he is old, and anticipates selling his invention at a license price of  $q$ . With a measure of entrepreneurs equal to  $(1 - \eta)r$ , he obtains  $(1 - \eta)rq$  for his invention, and cogitation is worthwhile only if:

$$(1 - \eta)rp(\sigma)q \geq A\theta. \tag{6}$$

The derivation of  $q$  is deferred to Section 2.3.2.

## 2.3 Markets and Prices

At date 1, all entrepreneurs possess the technology  $A$ , and the wage  $w$  and degree of specialization  $\sigma$  are determined in the labor market. Nerds then observe the production process, and if they have the incentives to cogitate, invent with probability  $p(\sigma)$ . At the second date, an innovation market as well as the labor market are active, and we think of them as opening in that sequence. Demand in both markets is generated by the entrepreneurs. The workers form the supply side of the labor market, while the old nerds who successfully invented when they were young supply the innovation market.

### 2.3.1 Labor Market

Since entrepreneurs who hire workers will always choose to do so at the maximum scale  $n$ , labor market equilibrium will generically involve one of only two levels of the wage share  $w$ . This will correspond to a case of excess supply of workers and a case of excess demand for them; in the latter situation, equilibrium requires an entrepreneur's indifference between hiring workers and operating the technology himself.

The labor market condition is reflected in the utility  $u^*$  that has to be guaranteed to a worker. Recall that the measure of entrepreneurs is  $r(1-\eta)$ , while that of workers is  $(1-r)(1-\eta)$ . If  $nr < 1-r$ , that is,  $r < \hat{r} \equiv \frac{1}{n+1}$ , supply exceeds demand, and the normal agents who are not hired obtain a payoff of zero. Since entrepreneurs can always find a worker who will accept any positive wage, they are not constrained; we can take  $u^* = 0$  and the equilibrium wage will be  $\underline{w}$ .

If  $r > \hat{r}$  when the labor market opens, the participation constraint in problem (5) binds, and  $u^*$  will be bid up until the potential entrepreneurs are indifferent between hiring workers and operating on their own: the corresponding wage share  $\bar{w}$ , with division of labor  $\underline{\sigma}$ , satisfies  $\pi(\bar{w}) = \alpha$ , since by operating on his own, an entrepreneur can get  $\alpha A$ . (In the nongeneric case in which the labor market is just balanced ( $r = \hat{r}$ ), any  $u^*$  corresponding to a wage share in  $[\underline{w}, \bar{w}]$  is consistent with market clearing.)

For later use, we denote the equilibrium wage when there is a measure  $r$  of rich normal by  $w(r)$ :  $w(r) = \underline{w}$  if  $r < \hat{r}$  and  $w(r) = \bar{w}$  if  $r > \hat{r}$ .

From the perspective of the entrepreneur, the lower output brought by specialization is the price to pay for maintaining a larger profit share. High degrees of specialization reduce aggregate output: a larger wage share would result in greater output, but smaller profits for entrepreneurs. However, from a dynamic perspec-

tive, specialization may enhance growth of aggregate output insofar as it facilitates invention.

### 2.3.2 Innovation Market

Suppose that a measure  $m < \eta$  of old nerds found ideas when young and now are bringing them to market. Each of these inventors has a monopoly on his idea and can offer it to all takers. Since improvements to each job enter symmetrically in the profit function, each entrepreneur cares only about the total number  $k$  of inventions he acquires, and wants at most one copy of each. The situation therefore conforms to a case of multiproduct monopoly in which each producer can offer his product at zero marginal cost.

Trade in the innovation market takes place as follows. (1) Simultaneously, each inventor sets a price for his idea; (2) each entrepreneur, taking these prices as given, chooses which inventions to purchase; (3) inventors incorporate their invention in the production process of the entrepreneurs who have agreed to purchase them.<sup>13</sup> An equilibrium of the innovation market is defined by a license price  $q_i$  for each inventor  $i$  and an adoption strategy  $k_l$  for each entrepreneur  $l$ . Taking the other inventors' prices  $q_{-i}$ , the labor market outside option of workers  $u^*$ , and the adoption strategies as given, an individual inventor does not want to modify his price. Taking the prices  $q_i$  and  $u^*$  as given, an entrepreneur does not want to modify his adoption strategy. We shall focus on symmetric (in license price) equilibria and assume that inventors cannot price discriminate among entrepreneurs (for instance, based on their future scale of operation).

If  $A$  is the level of technology that prevailed last period, the level for an entrepreneur who acquires  $k$  inventions is  $A(k) = A(1 + \gamma)^k$ , which is increasing and convex in  $k$ . Denote the payoff to an entrepreneur who adopts  $k$  inventions and faces outside option  $u^*$  by  $V(k, u^*)$ . The value of adopting  $k$  inventions is then  $V(k, u^*) - V(0, u^*)$ .

Now,  $V(k, u^*)$  incorporates the entrepreneur's scale decision (whether to operate as an artisan or hire workers) and the status of the participation constraint (whether it binds). The important property of  $V(k, u^*)$  is convexity in  $k$ , which it inherits from

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<sup>13</sup>For instance, the idea might be embodied in a part or equipment that the inventor installs. We want to avoid situations where the entrepreneur obtains the idea from an inventor and starts competing with him on the market for inventions. This possibility can only decrease the return from inventive activity and make growth more unlikely.

$A(k)$ : this is easy to see if the entrepreneur is an artisan ( $V(k, u^*) = \alpha A(k)$ ) or is unconstrained in the labor market ( $V(k, u^*) = A(k)\pi(\underline{w})$ ). In the Appendix, we show that convexity holds in the general case as well.

Convexity effectively weakens competition among inventors, since the marginal return from adoption is increasing. Assuming  $q_i = q$  in a symmetric equilibrium, the entrepreneur who maximizes  $V(k, u^*) - qk$  will choose  $k = m$ , i.e., buy all available inventions, as long as  $q \leq V_k(m, u^*)$  and  $V(m, u^*) - qm \geq V(0, u^*)$ . Putting  $q = \frac{V(k, u^*) - V(0, u^*)}{m}$ , which is less than  $V_k(m, u^*)$  by convexity, satisfies both of these conditions. (If  $q$  were lower than  $\frac{V(k, u^*) - V(0, u^*)}{m}$ , an inventor could increase his profit by raising his price a bit and the entrepreneurs would still purchase all of the inventions.) That this is the unique symmetric equilibrium follows from an argument in Tauman et al. (1997) in their analysis of multiproduct price competition. Notice that in this equilibrium, the inventors extract all of the surplus from the entrepreneurs.<sup>14</sup>

Write  $w(r)$  to denote the dependence of the equilibrium wage on the measure of entrepreneurs (from the discussion in Section 2.3.1, the wage is generically either  $\underline{w}$  or  $\bar{w}$ , and the normalized profit  $\pi(\underline{w})$  or  $\pi(\bar{w}) = \alpha$  depending on whether  $r < \hat{r}$  or  $r > \hat{r}$ ). The above discussion can be summarized in the following:

**Lemma 1.** (i) *Let  $m$  be the measure of inventions available at the beginning of the second period. In any symmetric equilibrium of the invention market all entrepreneurs purchase the  $m$  inventions.*

(ii) *There exists a unique symmetric price equilibrium:*

$$q(r, m) = \frac{\pi(w(r))(A(m) - A)}{m}.$$

Note that  $r > \hat{r}$  implies  $q(r, m) = \alpha[A(m) - A]/m$ , while if  $r < \hat{r}$ , we have  $q(r, m) = \pi(\underline{w})[A(m) - A]/m$ . Since  $[A(m) - A]/m = [(1 + \gamma)^m - 1]A/m$ , it is readily calculated that  $q(r, m)$  is an increasing function of  $m$  and that  $q(r, 0) = A\pi(w(r)) \log(1 + \gamma)$

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<sup>14</sup>In the finite case, there will typically be asymmetric equilibria as well, but in every case entrepreneurs purchase all innovations and the surplus is fully extracted by the inventors. Tauman et al. (1997) shows that the set of equilibria correspond to the core of cooperative game among the inventors, and we conjecture that as the number of goods gets large (our case) the set of equilibria “shrink” so that only the symmetric equilibrium remains in the limit.

### 2.3.3 General Equilibrium

We are now ready to determine the overall equilibrium of our economy by taking account of the nerds' cogitation decisions. The fact that the license price increases in the number of inventions leads to a strategic complementarity in cogitation. This raises the possibility of multiple (Pareto-ranked) equilibria, though we shall mainly be concerned with the Pareto optimal equilibrium and how its properties depend on the fundamentals of the economy.

Suppose that a fraction  $\phi \in (0, 1)$  of the nerds choose to cogitate at date 1. Then almost surely there will be  $\phi p(\sigma)\eta$  inventions on the market at date 2. This is an equilibrium only if  $p(\sigma(r))r(1 - \eta)q(r, \phi p(\sigma)\eta) = \theta A$  (if the left-hand side is less than the right, cogitating nerds would want to vegetate; if greater, the vegetators would want to cogitate). But since  $q(r, \phi p(\sigma)\eta)$  is increasing in  $\phi$ , a single vegetating nerd – there must be some, since  $\phi < 1$  – can gain by switching to cogitation (strictly speaking, this is not true in the continuum limit, but it is true of any finite economy that it approximates). Thus, the only possibilities for equilibrium are that all nerds cogitate or none do.

There is an equilibrium in which the innovation market is inactive if and only if

$$p(\sigma(r))r(1 - \eta)q(r, 0) < \theta A \quad (7)$$

and an equilibrium with an active innovation market if and only if:

$$p(\sigma(r))r(1 - \eta)q(r, p(\sigma(r))\eta) \geq \theta A. \quad (8)$$

If  $r$  is sufficiently small, (7) is satisfied, while (8) cannot be. Thus, when there are few entrepreneurs, the innovation market is inactive because the market for innovations is too small to encourage inventive activity.

What about larger values of  $r$ ? From our analysis of labor market equilibrium, there are generically only two values of the wage, division of labor, and profit level that concern us. Suppose that  $r < \hat{r}$ , so that  $w = \underline{w}$ . Then condition (8) is satisfied when  $r$  also exceeds a threshold value  $\underline{r} > 0$  satisfying:

$$\underline{r}(1 - \eta)p(\bar{\sigma})\pi(\underline{w})\frac{(1 + \gamma)^{p(\bar{\sigma})\eta} - 1}{p(\bar{\sigma})\eta} = \theta,$$

or

$$\underline{r} = \frac{\theta\eta}{(1-\eta)\pi(\underline{w})[(1+\gamma)^{p(\bar{\sigma})\eta} - 1]}.$$

Clearly, parameters can be chosen (in particular, let  $\theta$  be small) so that  $\underline{r} < \hat{r}$ . In this case, there is a general equilibrium of the economy in which  $(w, \sigma) = (\underline{w}, \bar{\sigma})$ , and the innovation market is active. The rate of technological improvement  $A'/A$  between dates 1 and 2 is  $(1+\gamma)^{p(\bar{\sigma})\eta}$ . If  $\underline{r} > \hat{r}$ , there are no values of  $r$  in which this high level of innovation can occur, since the innovation market is inactive for all  $r \in [0, \hat{r}]$ .

There is also a threshold value  $\tilde{r}$  of  $r$  below which the inactive innovation market equilibrium exists and above which it does not as long as the wage is  $\underline{w}$ . It is straightforward to check that  $\tilde{r} > \underline{r}$  and that  $\tilde{r} < \hat{r}$  for appropriate choice of parameters. Thus there is a nonempty set of  $r$  values in which the two equilibria co-exist. The cogitation equilibrium Pareto dominates the vegetating equilibrium: nerds and date 2 workers benefit from the technological improvements, though (date 1) workers and all entrepreneurs are indifferent.

Since we are interested in the possibility of growth, and since multiple equilibria of this kind have been dealt with elsewhere in the literature on growth and development (e.g., Murphy et al. 1989, Grief 1994, Mokyr 2005), we shall focus on the Pareto optimal equilibrium, except for a brief discussion in the Conclusion.

For the case  $r > \hat{r}$ , a similar argument establishes the existence of a threshold  $\bar{r} = \frac{\theta\eta}{(1-\eta)\pi(\bar{w})[(1+\gamma)^{p(\underline{\sigma})\eta} - 1]}$  above which the innovation market is active. Of course, if  $p(\underline{\sigma})$  is small enough,  $\bar{r}$  may exceed 1, so that there is never innovation in the “artisanal” labor market equilibrium. More generally, even if the innovation market is active, the rate of innovation is lower than it is in the factory equilibrium ( $r < \hat{r}$ ) than in the artisanal equilibrium. (There is also a corresponding  $\bar{\bar{r}} > \bar{r}$ , below which the inactive innovation market equilibrium exists, though this shall not play much role in our analysis.)

The forgoing analysis underscores the interaction between the markets for innovations and for workers. When there is excess supply of workers, the wage is small and specialization is high, and the arrival rate of ideas is high. Many inventions are offered on the market and by convexity of their value to entrepreneurs, the price of a license is high. This would suggest that nerds indeed have strong incentives to search for inventions. However, if  $r$  is too small the revenue  $rq$  may be so small that the expected return from invention is small compared to its cost.

When there is excess demand of workers, there are many entrepreneurs who could

pay for innovations. However, specialization is low and since the probability of discovery is now small, there can be only a few inventors active on the invention market. The price of the license will be small, both because entrepreneurial profits are small and because there are fewer innovations. Hence incentives to invent are small in this case.

Therefore, the incentives to invent are small when there are too few or too many entrepreneurs: in the first case there is not enough demand for innovation to cover its cost, while in the second case, high wage shares and low specialization make invention less probable as well as less remunerative. It is only in the intermediate range that high rates of innovation will happen: both the demand for inventions and the probability of discovery are high.

We summarize this discussion with the main result of this section.

**Proposition 1.** *Let  $\underline{r} < \hat{r}$ .*

- (i) *If  $r \in [0, \underline{r}]$ , the equilibrium labor contract is  $(\underline{w}, \bar{\sigma})$  but there is no innovation.*
- (ii) *If  $r \in [\underline{r}, \hat{r}]$ , there is an equilibrium with labor contract  $(\underline{w}, \bar{\sigma})$  and an active innovation market with technological improvement rate  $\bar{g} \equiv (1 + \gamma)^{p(\bar{\sigma})\eta}$ .*
- (iii) *If  $r > \hat{r}$ , the equilibrium labor contract is  $(\bar{w}, \underline{\sigma})$ . For  $r \in (\hat{r}, \bar{r})$  there is no innovation; for  $r \in [\bar{r}, 1]$  the improvement rate is  $\underline{g} \equiv (1 + \gamma)^{p(\underline{\sigma})\eta}$*

Note that it is possible that one of the intervals in part (iii) is empty; either way, the growth rate of technology is non-monotonic as  $r$  varies over  $[0, 1]$ . There can be both too much as well as too little entrepreneurship (as measured by  $r$ ) for innovation. When there are many entrepreneurs, individuals work in firms with little labor division, similar to artisanal systems of production. Few resources (here measured by  $c\sigma$ ) are wasted in supervision. In this sense the economy is *statically efficient*, since output per capita is high relative to the state of technology. But it is *dynamically inefficient* since it produces innovations at a low rate, and technology is likely to be relatively backward.

The non-monotonic relationship between inequality and growth is driven by the interplay between entrepreneurial incentives to divide labor, which happens when inequality is high and wages are low, and the nerds' incentives to innovate, which is high when entrepreneurs are numerous because their collective profit is large. The second is a market size effect and is a consequence of our assumption that each entrepreneur may hire up to  $n$  workers and therefore effectively faces a "capacity" constraint. Little would change with a more flexible production technology that

allows entrepreneurs to hire any number of workers as long as it displays sufficient (eventual) diminishing returns to scale.<sup>15</sup> The latter could come from a number of sources: beside the standard technological origins (each entrepreneur has limited time or attention to market goods, organize production, etc.), financial contracting problems could limit scale if  $r$  is interpreted as a fraction of the population with sufficient wealth (see next section) and production requires capital as well as labor.<sup>16</sup>

Smith famously argued, in what many have interpreted as self-contradiction, that the high level of specialization he observed in factories was counterproductive, requiring government intervention: the worker “becomes as stupid and ignorant as it is possible for a human creature to become.” Viewed from the perspective of the present model, there is no contradiction: it is the nerds, not the workers, who generate the ideas. More generally, in a world of externalities, such as the one depicted here, this is exactly what one might expect: the equilibrium degree of labor division may well be too high, likely in the range of negative marginal productivity returns, because of the monitoring benefit. Even if workers were responsible for inventing (so  $p(\sigma)$  might be decreasing for high levels of  $\sigma$ ), entrepreneurs would have little reason to fully internalize the effects of labor division on worker ignorance, offering another reason why too much inequality would harm the rate innovation.

### 3 Dynamics

In this section we extend our model by endogenizing the occupational distribution and illustrating that the three regimes discussed in Proposition 1 can be steady states. The model displays endogenous growth; the novelty here is that growth is driven by organizational design rather than a technical progress production function.

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<sup>15</sup>In the usual competitive model in which output per entrepreneur is given by a smooth production function  $F(\ell)$ , where  $\ell$  is the number of workers hired, and  $F(\cdot)$  satisfies standard properties, in particular the “Inada” condition  $\lim_{\ell \rightarrow \infty} F'(\ell) = 0$ , aggregate profit is  $r[F(\ell) - w\ell]$ , where  $w$  is the equilibrium wage. Since  $\ell r = 1 - r$  and  $F'(\ell) = w$  in equilibrium (here we ignore the worker incentive problem, which doesn’t affect the argument), the aggregate profit is  $rF(\frac{1-r}{r}) - (1-r)F'(\frac{1-r}{r})$ , which is bounded above by  $rF(\frac{1}{r})$ . It is straightforward to see (use l’Hôpital’s rule and the Inada condition) that the latter expression converges to zero as  $r \rightarrow 0$ .

<sup>16</sup>With a few large entrepreneurs, the monopoly power of inventors would plausibly be diminished as well, which would decrease the share of profits they can extract, further reducing their cogitation incentives.

### 3.1 The Dynamic Model

Consider the above economy repeating itself infinitely often. In each period  $t = 1, 2, \dots$  every one of the continuum of individuals gives birth to one offspring; with probability  $\eta$ , independent across lineages and periods, the child is a nerd; otherwise he is normal. Normalize the size of the population born at each period to be unity.

All individuals live for two periods and consume only in old age, when they also give birth. Normal individuals are idle in youth and active (as workers or entrepreneurs) in old age. Nerds are active in youth and, once they have cashed in on their inventions, idle in old age. Individuals born at time  $t$  have preferences characterized by the utility

$$U^t(c^t, b^t) = \gamma c_t^{1-\beta} b_t^\beta,$$

where  $c_t$  is generation  $t$  consumption,  $b_t$  is a monetary investment made by the parent in the child's human capital,  $1 > \beta > 0$ , and  $\gamma = \beta^{-\beta}(1 - \beta)^{\beta-1}$ . Indirect utility is therefore equal to the net lifetime income  $y_t$ , and the investment is  $\beta y_t$ .

The key assumption is that there is a credit market imperfection – the parental investment effectively determines the (normal) child's occupation. We model this by supposing that there is a minimum threshold investment  $hA_t$  for access into entrepreneurship. This may be interpreted as the cost of sufficient education, of a set of contacts, or even the physical capital to set the child up in business for himself, as long as it is unaffected by technological improvements. Then its cost rises with the general level of technology.<sup>17</sup>

Finally, we assume that the improved technology  $A'_t$  from one generation diffuses completely to become the current technology  $A_{t+1}$  for the next.

We continue to assume  $\underline{r} < \hat{r}$ . To keep the analysis as simple as possible, we will impose some additional conditions on the parameters. First, we shall assume that  $p(\underline{\sigma})$  is small enough that  $\bar{r} > 1$ . Next, we impose:

**Assumption 2.**  $\alpha > \bar{w} \bar{g}$ .

This implies that entrepreneurship, even at its smallest scale, is preferable to working, even at the highest possible equilibrium wage; that way, in any equilibrium,

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<sup>17</sup>This sort of assumption has appeared elsewhere in the literature on growth with credit constraints (e.g., Mookerjee and Ray 2002). While education or contacts are clear candidates to satisfy the requirement, so would certain types of physical capital: there was, for instance, little technological change in building construction over the course of the industrial revolution (Pollard 1965, p. 84-85).

agents whose parents set them up in business actually want to remain entrepreneurs. (Recall that if there is innovation in artisanal equilibrium, an entrepreneur’s income is  $\alpha A$ , while a worker gets  $\bar{w}A'$ ;  $A'/A$  may be as high as  $\bar{g}$  because the growth rate is determined in the *previous* period by the degree of specialization that prevailed then.) Alternatively, one could suppose that there is a high enough “private benefit” to entrepreneurship.

**Assumption 3.** (i)  $\beta \underline{w} < h$  and  $\beta \theta < h$ : children of low wage workers and of vegetating nerds cannot access entrepreneurship.  
(ii)  $\beta \bar{w} > h$  and  $\beta \theta / p(\bar{\sigma}) \geq h \bar{g}$ : normal children of high wage workers and normal children of inventors can be entrepreneurs.

These conditions avoid trivialities: if children of low wage workers are not wealth constrained, all agents are rich after the first generation; if children of high wage workers and of inventors are wealth constrained, then the proportion of rich in the economy declines and the economy always ends up at subsistence. (The second part of (ii) is equivalent to  $\beta(1 - \eta) \underline{r} q(\underline{r}, p(\bar{\sigma}) \eta) > h A'$ , which is the condition that inventors in the factory equilibrium can afford entrepreneurship for their children; it implies that inventors in the artisanal equilibrium can afford it as well.)

Nothing of significance turns on Assumption 2 or its alternative; if it is violated, the analysis of equilibrium in case of excess labor demand becomes slightly more cumbersome because the “ex-post” indifference of entrepreneurs between hiring workers and operating on their own must be replaced by “ex-ante” indifference between entrepreneurship and working, taking account of innovation costs; this forces the labor market into balance ex-post, lowers the highest equilibrium wage somewhat, and eliminates the possibility that entrepreneurs operate at small scale. The qualitative relationship between wealth distribution and innovation remains unchanged.

Similarly, neither the form of the parental investment motive (here it is of the “warm glow” variety) nor the fact that the parent invests directly rather than transferring cash to the child is important here. Other bequest motives would also have the threshold effects we will be exploiting below. So would allowing the child to take account of the investment cost in choosing occupations. In both cases the analysis would be slightly more complicated, raising similar issues to those raised by violations of Assumption 2.

## 3.2 Analysis

The important point to note is that if investment is less than  $h$ , the normal child is a worker (the nerd does not need the investment, and it would make no difference to the analysis if we simply assumed that his parent didn't invest for him at all). We shall call children whose parents invest more than  $h$  "rich" and the rest "poor". Thus, the children of (successful) inventors, entrepreneurs, and workers when the wage is high are rich.

The state variable is  $r_t$ . The above assumptions imply the following.

**Lemma 2.**

- (i) If  $r_t < \underline{r}$ , then  $r_{t+1} = (1 - \eta)r_t$
- (ii) If  $\underline{r} \leq r_t < \hat{r}$ , then  $r_{t+1} = (1 - \eta)r_t + \eta p(\bar{\sigma})$
- (iii) If  $r_t > \hat{r}$ ,  $r_t = 1 - \eta$

*Proof.* (i) There is excess supply but the condition for an operative invention market is not satisfied; workers get a low wage  $\underline{w}$  and their children cannot be entrepreneurs. The only ones who can are the offspring of entrepreneurs: since their profits exceed  $\bar{w}$ , by Assumption 3 they can invest  $hA_t$ .

(ii) There is excess supply, hence the wage is  $\underline{w}$ , but now the invention market is operative. As before, normal children of entrepreneurs can be entrepreneurs, and there are  $(1 - \eta)r_t$  of them. By Assumption 3 the  $\eta p(\bar{\sigma})$  inventors will invest enough to give their normal children access to entrepreneurship.

(iii) There is excess demand, hence high wages, and all children of normals can become entrepreneurs. □

There are two or three steady states, depending on whether the fixpoint  $r^*$  ( $= p(\bar{\sigma})$ ) of  $(1 - \eta)r_{t-1} + \eta p(\bar{\sigma})$  lies in  $[\underline{r}, \hat{r}]$ . In either case, all steady states are locally stable.

If  $r^* < \underline{r}$ , we are in the "dismal case": the two steady states are  $r = 0$  and  $r = 1 - \eta$ : both cases are incompatible with growth. When  $r = 0$ , the equilibrium is an economy of pure subsistence where each individual produce  $A\mu$ . The case  $r = 1 - \eta$  by contrast is a statically prosperous economy in which almost everyone invest and become a small scale entrepreneur, this implies that there is stagnation because of the low degree of specialization. Subsistence eventually occurs if the economy starts below  $\underline{r}$ ; stagnant prosperity results if it begins above  $\underline{r}$ . There might however, be a short period of innovation in case the economy happens to start in  $[\underline{r}, \hat{r}]$ , but collapse into subsistence soon follows. See Figure 1.

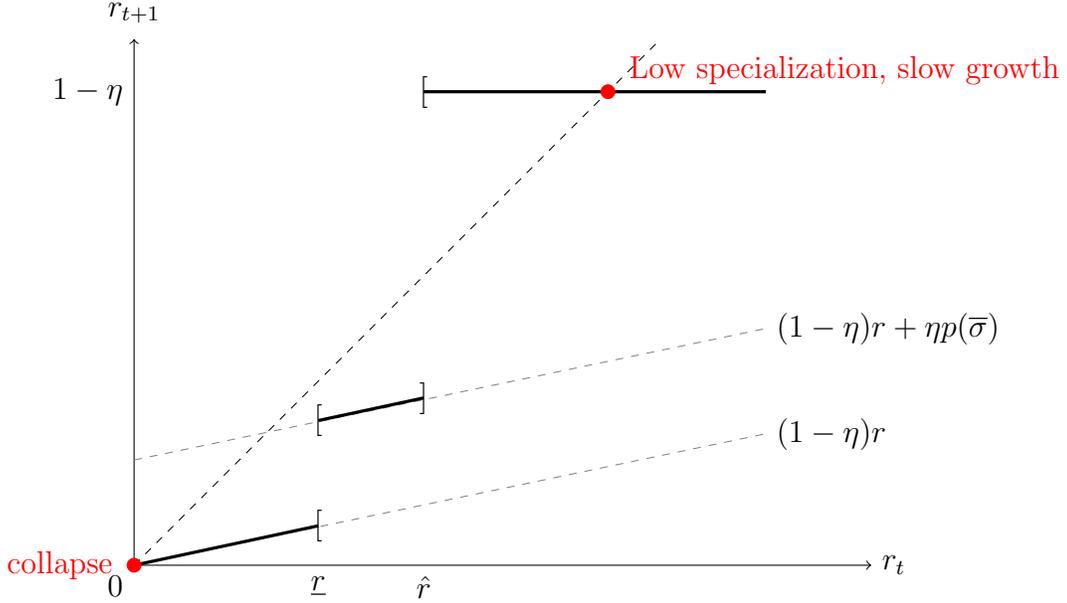


Figure 1: The Dismal Case ( $\underline{r} > r^*$ ): Slow or No Growth

A slightly less dismal case occurs if  $r^* > \hat{r}$ ; in this case  $p(\bar{\sigma})$  is relatively large, and the  $(1 - \eta)r_{t-1} + \eta p(\bar{\sigma})$  branch lies above the 45°-line. Permanent high growth is not possible, though again the economy may experience growth for a few periods. The basin of attraction for subsistence is smaller than in the dismal case, consisting only of the interval  $[0, \underline{r})$ .

The case of greatest interest is the “hopeful” one in which  $\underline{r} \leq r^* < \hat{r}$ , in which there is another locally stable steady state at  $r^*$ . Any economy beginning in the interval  $[\underline{r}, \hat{r})$  converges to  $r^*$ . Here the wage share is low and there is a high degree of division of labor, and a technological growth rate of  $(1 + \gamma)^{\eta p(\bar{\sigma})}$  (see Figure 2).

(If  $\underline{r} > \hat{r}$ , then we are in a truly dismal case where the innovation market is *never* operative and the economy proceeds either to subsistence or to prosperous stagnation.)

Note except in the nongeneric case that  $r^* = \hat{r}$ , the economy cannot spend any time at  $\hat{r}$  unless it happens to start there, so that we are justified in ignoring the cases of intermediate wages.

The hopeful case depicted in Figure 2 is the one that suggests the possibility of an inverse U relation between the degree of inequality (measured by  $1/r$ ) and the rate of growth: economies with *either* high or low degrees of inequality (low or high  $r$ )

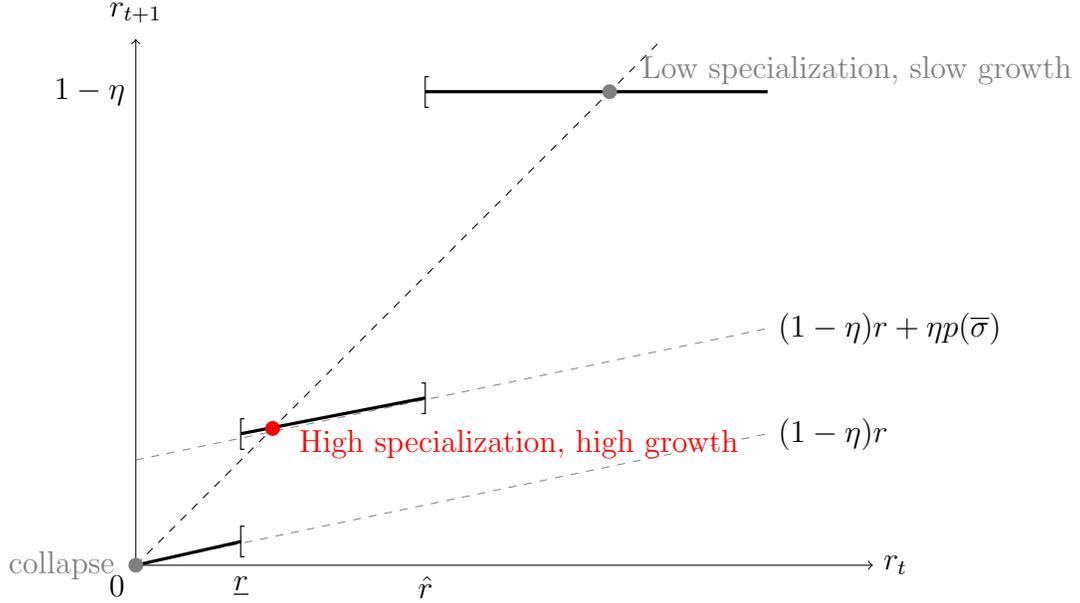


Figure 2: The Hopeful Case ( $\underline{r} < r^* < \hat{r}$ ): High Growth

grow slowly or not at all, while those with middling levels are the ones that generate sustained technical progress.

The forgoing discussion can be summarized in the following

**Proposition 2.** *Let  $\underline{r} < r^* < \hat{r}$ , and the fraction of rich at  $t = 0$  be  $r_0$ .*

(i) *If  $r_0 \in [0, \underline{r})$ , the sequence  $\{r_t\}$  converges to the subsistence steady state with zero growth.*

(ii) *If  $r_0 \in [\underline{r}, \hat{r})$ , the sequence  $\{r_t\}$  converges to  $r^*$  and the economy has a steady-state growth rate of  $\bar{g} \equiv (1 + \gamma)^{\rho(\bar{\sigma})\eta}$ .*

(iii) *If  $r_0 > \hat{r}$ , the sequence  $\{r_t\}$  converges to  $1 - \eta$  and there is zero growth*

Note that (iii) depends on the simplifying assumption we made that  $\bar{r} > 1$ . If  $\bar{r} \leq 1 - \eta$ , we are in the (possibly more plausible) situation of low ( $\underline{g}$ ) but positive steady-state growth.

## 4 Comparative Dynamics

Here we consider three types of changes: improvement in the access to capital; an increase in technological complexity; improvement in organizational innovations that reduce coordination costs.

## 4.1 Institutional Improvements in the Access to Entrepreneurship

We model this as a reduction in  $h$  necessary to run a business, a change that increases the fraction of the population that can afford to become entrepreneurs. This might come from improvements in credit markets, or if  $h$  is interpreted as a human capital acquisition cost, from education subsidies. It even come from loosening the kinds of legal restrictions, common in many developing countries, that generally limit business startups (typically these bind on the less wealthy).

For a formal treatment, it is helpful to give the economy *some* chance of emerging from a subsistence steady state if only  $h$  were low enough. So assume that subsistence generates positive earnings  $sA$ , with  $s < \underline{w}$ , and that  $\beta$  is a random variable, independent across generations and lineages, and independent of income or nerdiness. Specifically,  $\beta = \bar{\beta}$  with probability  $b$ , and  $\underline{\beta} < \bar{\beta}$  otherwise. Let  $\underline{\beta}$  satisfy all of the conditions in Assumption 3. Notice that this modification to the model doesn't change the values of  $\underline{r}$  and  $\hat{r}$ .

In addition, assume that initially, we have  $h > \bar{\beta}\underline{w}$ , so that the economy behaves just as it did before. In particular, the subsistence steady state  $r = 0$  always exists and is locally stable, though now people are investing  $\beta s$  in their children.

Now let  $h$  fall to a point where  $\bar{\beta}s > h > \underline{\beta}s$ , and suppose that  $\underline{r} < b < \hat{r}$ . If we started at the subsistence steady state, the effect of this decline in  $h$  is to increase the value of  $r$  next period, from 0 to  $b$ . But this means we have  $(1-\eta)b$  entrepreneurs, who hire  $(1-\eta)nb$  workers at the wage  $\underline{w}$ . Since  $\underline{r} < b$ , the innovation market begins to function (assuming the nerds coordinate on the cogitation equilibrium). The children of the entrepreneurs and inventors, as well as the children of the generous ( $\bar{\beta}$ ) workers, will become entrepreneurs next period.

Locally, the dynamics are now following the equation  $r_{t+1} = (1-\eta)r_t + p(\bar{\sigma})\eta + b(1-\eta)(1-r_t)$ , which converges to a new steady state  $r^{**} = \frac{(1-\eta)b + \eta p(\bar{\sigma})}{\eta + b}$ . For plausible parameter values ( $\eta$  small), this exceeds  $r^*$ , but remains in the factory equilibrium region (less than  $\hat{r}$ ) if  $b$  is small. Thus, as expected, increasing access to entrepreneurship can pull the economy out of subsistence and onto the path of industrialization and high steady-state growth.

But, for other parameters (larger  $b$ ), the economy may eventually leave the interval  $[\underline{r}, \hat{r}]$ , and the growth process slows down. Similarly, further reductions in  $h$  may turn out to be too much of a good thing: if  $\beta\underline{w} > h$ , then all normals set their children up

in business, and innovation ceases or at best declines.

Thus, “improvements in institutions” may have ambiguous effects, depending on where the economy is to begin with. An economy that has very poorly functioning credit markets or costly education will generally be helped by improvements in these institutions. But economies in which these institutions are functioning moderately well may actually be hurt.

If one were to measure the rate of TFP growth across economies with different qualities of institutions, one may therefore find that growth rates are not monotonic in the quality measure. Similarly, since the levels TFP will depend on the history of their growth, neither should there be any expectation of finding a monotonic pattern in a cross-country regression of TFP levels on institutional quality.

## 4.2 Geographical Linkages and the Spread of Industrial Revolution

A natural question to ask is whether integration of geographic regions might enhance the chances for a successful takeoff: if one region happens to enter the high innovation steady state, perhaps it can stimulate others to do the same. For definiteness, assume the “hopeful case” parameter configuration discussed in the previous section prevails, and suppose there are two equally populated regions, 1 and 2, identical in all respects except for their wealth distributions. One has an unequal distribution with  $r_1 \in [\underline{r}, \hat{r}]$  and the other has a more equal distribution  $r_2 > \hat{r}$ . Without linkages, region 1 is growing while region 2 is not.

A somewhat trivial answer to our question can be obtained if one assumes that innovations can diffuse (with a lag) across regions, just as we have assumed for the baseline dynamics in the previous section, but there are no factor flows (in particular of nerds or normals) across borders. In this case, invention continues to be confined to region 1, but region 2 benefits by implementing the new ideas in the ensuing periods as they become publicly available. In steady state, consumption grows at the same rate in both regions, but region 2 will always be at a lower level, equal (at best) to the region 1 consumption of the previous period.

Of course, the assumption of complete and free diffusion is somewhat facile, made here to maximize the chances for sustained growth; indeed adopting existing technologies is rarely “easy” (see for example Acemoglu et al. 2006); if it were, there would be hardly be a problem of underdevelopment! Indeed, one interpretation of micro-

inventions is that they partly involve adapting production processes or products to local conditions and markets. So let us ask whether regional interaction will allow region 2 to become a “cradle of innovation” once the process has started in region 1.

At the other extreme, we could assume complete integration of the two regions: nerds and normals are free to move, so here is one large economy. There are two effects here. First, there is a redistribution of wealth (change in  $r$ ): let the wealth distribution  $r'$  for the united region to be the average of the two distributions we began with:  $r' = \frac{1}{2}(r_1 + r_2)$ . Second, because the size of the entrepreneurial population has increased, the incentives to cogitate are strengthened. Condition (8) now becomes

$$p(\sigma(r))2r(1 - \eta)q(r, p(\sigma(r))2\eta) \geq \theta A, \quad (9)$$

because the population has doubled. Since  $q(r, m) = \pi(w(r)) \frac{(1+\gamma)^m - 1}{m}$ , the left hand side of (9) increases, given  $r$ . It follows that  $\underline{r}$  decreases, so that the basin of attraction for innovation increases in size. (A similar calculation shows that  $\bar{r}$  decreases as well, so that the basin of attraction for moderate growth may also increase.) The recursion equations in Lemma 2 are otherwise unchanged.

The increase in the basin of attraction would seem to be good news for the spread of the industrial revolution. (To be sure, it is predicated on the complementarities among inventions and the lack of any possibility of duplication despite the greater population of nerds; if duplication were possible, incentives to innovate would be weakened somewhat, and there would be less clear effects on the basin of attraction for high growth.) However, whether the new integrated region will continue to grow depends on how the change in dynamics interacts with the change in initial conditions brought about by mixing the wealth distributions. Indeed, in the case we have been considering, in which  $r'$  is between  $r_1$  and  $r_2$ , integration will lead to convergence to the high growth steady state only if  $r' < \hat{r}$ . If instead  $r' > \hat{r}$ , then the integrated economy converges to the prosperous stagnation steady state, and the industrial revolution ends. Of course other cases are possible. A growing region could integrate with a very unequal (low  $r$ ) region, leading either to overall growth or to stagnation. Or an unequal region and a prosperously stagnant one could integrate, and the industrial revolution could take off. On balance, the distributional effects of regional integration make the process of takeoff only slightly less difficult.<sup>18</sup>

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<sup>18</sup>An intermediate case of integration is to restrict mobility across regions to the nerds. This introduces a more subtle analysis that is beyond the scope of this paper; the bottom line is that

### 4.3 Technological Complexity and Modern Developing Economies

To what extent can Smithian growth facilitate development in the modern world? On the one hand, a poor (low  $r$ ) country's entrepreneurs may have a higher willingness to pay as soon as they are able to sell on a world market, and this may incentivize the local nerds, allowing for an expanded basin of attraction similar to the case discussed above.

On the other hand, the relevant technological frontier may be further away (Acemoglu et al. 2006). One way to model this is think in terms of the level of complexity, represented by the task to population ratio; call it  $\chi$  (thus far, it has been assumed equal to one). The production technology becomes  $\exp(\int_0^\chi \log[a_t(j)l_t(j)]dj)$ . The growth rate is now  $(1 + \gamma)^{\eta p/\chi}$ , and the threshold value  $\underline{r}$  of educated that sustains growth is  $\frac{\theta}{(1-\eta)[(1+\gamma)^{\eta p/\chi}-1]\pi(\underline{w})}$ . With simple technologies ( $\chi$  small), growth rates sustained by the Smithian mechanism are high, and the economy is more likely be able to sustain growth because  $\underline{r}$  is small and  $\hat{r}$  is unchanged.<sup>19</sup> An exogenous increase in the complexity of technology will lower growth rates and increase the likelihood that technological progress comes to a halt.

If one interprets complexity as originating in a world technological frontier, one that may be expanding through “internalized” R&D, an implication of this observation is that Smithian growth may be a particularly inadequate mechanism by which a developing country might catch up to the rest of the world. Although an increase in  $\chi$  instantaneously increases output (assuming the  $a(j)$  of the “new” steps are equal on average to the rest), over the longer run, this benefit would be overshadowed by the slowdown in growth, and a Smithian country will fall further and further behind the R&D-driven frontier. Possibly more than their more developed counterparts, such countries may have to rely on non-Smithian mechanisms (state subsidies, or private-sector internalized R&D, both of which became more or less common in the rich countries in the latter half of the 20th century) to achieve innovation and growth.

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there is a tendency for innovation to increase relative to the no-mobility case, but to be confined to only one region.

<sup>19</sup>It may be reasonable to suppose that increased complexity would increase the coordination cost for a given choice of  $\sigma$ , since there are now more tasks to coordinate. For instance the coordination parameter cost might be written as  $c\chi$ . All else the same, the division of labor would be reduced with increased complexity and the effects on growth exacerbated.

## 4.4 Organizational Innovation: Revisiting the Role of the Factory in the Industrial Revolution

As suggested earlier, a decrease in the coordination cost  $c$  can be identified with the introduction of the manufactory and subsequently the early factories. When  $c$  falls, the division of labor becomes less costly for entrepreneurs. Moreover,  $\underline{w}$  decreases,  $\bar{\sigma}$  increases and therefore  $\underline{r}$  falls while  $r^*$  rises. Hence the basin of attraction of the high growth steady state is enlarged, firms have a larger division of labor there, and the growth rate increases. There is also more income inequality in the functional distribution of income and a likely increase in inequality in the size distribution (since  $r^* < \hat{r}$ , increasing the fraction of rich slightly raises most inequality measures).

Observe that that for  $r > \hat{r}$ , since  $\bar{w}$  increases,  $\underline{\sigma}$  falls. Therefore the rate of growth decreases in this region. Hence, in our model, organizational innovations leading to a decrease in the cost of coordination tend to intensify the dependence of growth on inequality. Once the manufactory comes into being, changes in inequality would lead to larger changes in growth rates than they would have under the putting out system.

Our model suggests that the significant role played by the factory and the introduction of the division of labor has to do with the concomitant increase in labor division that would have facilitated invention, rather than any static increase in surplus that factory production might have generated. Indeed, in our model, the factory ( $\sigma$  large) actually generates *less* surplus than does “artisanal” production. Some historians (e.g., Clark 1994, Cohen 1981) have argued similarly that in the early years of the Industrial Revolution, the factory was perhaps more profitable but not more productive than the putting out system it replaced.

## 5 Conclusion

We have explored a causal link between the organization of firms and technical progress. In contrast to such links that have been explored so far in organizational growth theory, the mechanism we study involves an externality in the benefits of invention, rather than on firms’ internalization of those benefits, and is arguably especially pertinent to understanding early episodes of growth such as the Industrial Revolution. Starting from the “Smithian” idea that there is an increased likelihood of innovations in the production process when labor is more specialized, we show that entrepreneurs may be induced to choose innovation-enhancing organizations even though intractable

contractual incompleteness and incentive problems prevent them from appropriating the returns to innovations. The conditions that do this in a laissez-faire market equilibrium depend on a constellation of factors: a free enterprise legal environment that allows an individual with an idea to sell it to a sufficiently large fraction of the market; an imperfect credit market that restricts entry into entrepreneurship; a distribution of wealth that is neither too equal nor too unequal; and the need of a coordination device among inventors.

An economy that generates technological progress initially may eventually violate the distributional condition, or may, for reasons having to do with improvements in credit markets, subsidized education, or other redistributions, switch to a no progress equilibrium, with firms that are too unspecialized to foster further Smithian innovation: “trickle down” effects may eventually limit growth.

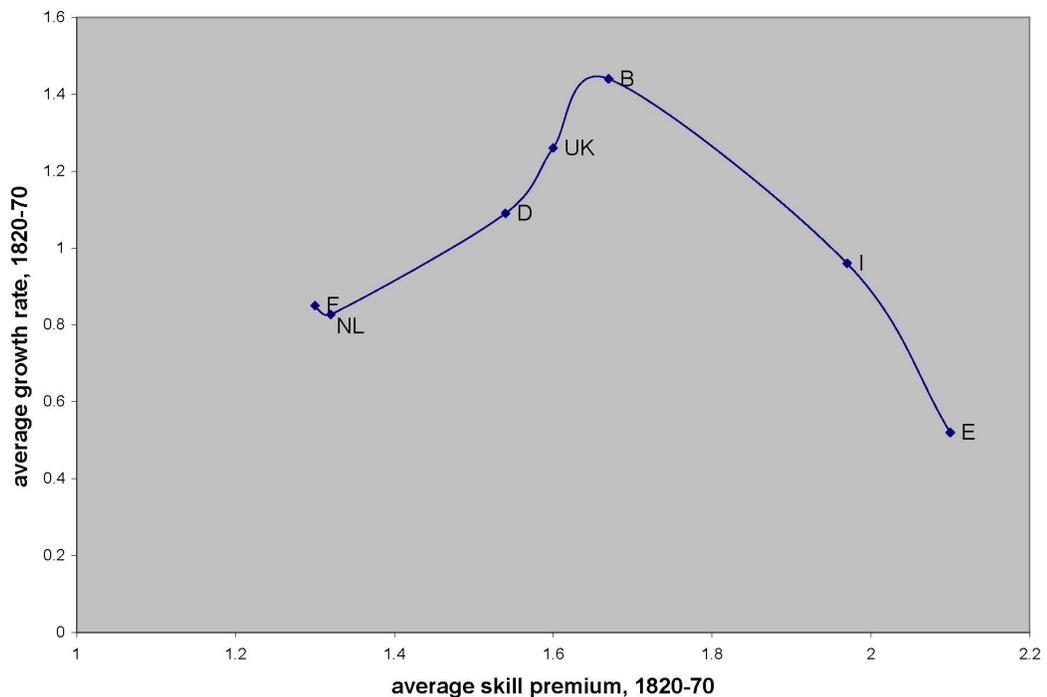


Figure 3: Inequality versus Growth

The chart in Figure 3 plots average growth rates of per capita GDP against inequality for several European countries for the period 1820-1870. Growth rates are from Maddison (2001), and inequality is measured as the ratio of skilled to unskilled wages, taken from Allen (2005).<sup>20</sup> The inverted U pattern is clearly displayed, with growth rates for the lowest skill premium countries (Netherlands and France) slightly higher than those for the highest skill premium country (Spain). The contrast between the Netherlands, for which the wage data are from (Protestant) Amsterdam, and Belgium, for which wages are from (Catholic) Antwerp, is also striking. Obviously evidence of this sort is at best indicative (for instance, institutions are not identical across countries, and we would rather have TFP growth data than GDP growth), but it does accord broadly with the basic predictions of our model.<sup>21</sup>

Furthermore, the model has other implications for growth theory more generally besides the link it draws between inequality and technological progress. First, “institutional” improvements, such as the increased efficiency of credit markets need not have monotonic effects on the rate of technological progress. Thus, starting from very poorly functioning markets, both static and dynamic efficiency are likely to improve as output increases and the demand for inventions increases enough to activate the innovation market. But further improvements to these institutions will eventually reduce the division of labor and therefore the rate of technical progress and economic growth. An economy with moderately well functioning credit markets that has been rapidly growing a while will have higher productivity than one with perfect credit markets that has been growing slowly or not at all. It follows that total factor productivity need not be monotonic in the “quality of institutions,” either over time or in cross section.<sup>22</sup>

Second, taking the (manu)factory to be the organizational innovation that reduced the cost of labor division, then one interpretation of the forgoing is that organiza-

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<sup>20</sup>Since Allen’s wage data are for Milan, “Italy” is actually Lombardy 1836-1857, with the growth taken from Pichler (2001) and calibrated to Maddison’s other estimates using the two authors’ estimates for Austria (Maddison’s estimate for all of Italy is somewhat lower than our imputed estimate for Lombardy; this has little effect on the basic pattern).

<sup>21</sup>So does other recent historical scholarship on the industrial revolution in Britain (e.g., Crafts and Harley 2002; O’Brien 1986, p. 297). This research emphasizes the peculiar roles of the enclosure movement in creating a large population of landless poor to the supply the labor markets and in helping to generate a relatively large upper-middle class and concomitantly large market for manufactured goods. Lacking these elements, other European countries were slower to industrialize.

<sup>22</sup>The model thereby also offers a mechanism for “reversal of fortune” phenomena that have been documented in historical cross-country comparisons of economic prosperity (e.g., Acemoglu et al., 2002).

tional improvements may indeed have helped to lead the economy from a path of subsistence to one of sustained growth, not so much by reducing the cost of entry into entrepreneurship, but more importantly by raising the equilibrium level of specialization, thereby facilitating invention. This seems to support the “institutional” view of the importance of the factory system in the industrial revolution, albeit perhaps not in the manner Cohen (1981), Millward (1981), and North (1981) argued.

Finally, it is worth remarking on the implications of the model in case of coordination failure among the inventors. For the high-wage share regime, it makes only a small difference. If there was a switch to the no-invention equilibrium, the fraction of rich would fall, as would the growth rate, albeit not by much since it was low to begin with: the new steady state would be  $r = 1 - \eta$  instead of  $r = 1 - \eta + \eta p(\underline{\sigma})$ .

On the other hand, the same switch from cogitation to vegetating equilibrium in the low-wage share regime would be more dramatic. An economy in the basin of attraction of the high-growth steady state now follows the “dismal” dynamics  $r_{t+1} = (1 - \eta)r_t$ . In short order, the economy would be carried below  $\underline{r}_C$ , outside of the basin of attraction of high growth, and outside of the region where the cogitation equilibrium exists.

This analysis offers a specific interpretation to the view, expressed by some historians and economists (e.g., Crafts, 1985; Acemoglu and Zilibotti, 1997), that the Industrial Revolution occurred because of “luck.” Many factors, partly institutional, partly technological, and partly distributional, must fall into place in order for a period of sustained technological growth to emerge. As our model suggests, the path to sustained prosperity is a narrow one, difficult to find, and easily lost.

## 6 Appendix

### 6.1 Proof of Lemma 1 - The Symmetric Case

We establish the Lemma in a finite economy, that is when there are finitely many inventors and agents. The continuum economy can be approximated by arbitrarily large such finite economies and we restrict attention to equilibria of the continuum economy that are the limits of equilibria with finite economies.

There are  $m$  inventions available. We consider symmetric price equilibria, that is when all inventors post the same price  $q$  in equilibrium.

**Case 1: Excess supply for labor** When there is excess demand for labor

( $r < \hat{r}$ ), entrepreneurs are not constrained since the outside options of workers is  $u^* = 0$  and they will offer a wage  $\underline{w}A(k)$  independently of the technology they have adopted. On the innovation market, faced with prices  $q$  for each innovation they solve

$$\max_k \pi(\underline{w})A(k) - kq$$

Because  $A(k)$  is convex increasing,<sup>23</sup> it is immediate that conditionally on adopting, entrepreneurs will choose  $k = m$ , and their payoff from adopting is then  $\pi(\underline{w})A(m) - mq$ . They will adopt only if  $\pi(\underline{w})A(m) - mq \geq \pi(\underline{w})A(0)$ . If the inequality is strict, one of the inventors could deviate to a higher price and increase his profit. Clearly there is no incentive for an inventor to decrease his price since he would face the same demand for his invention. Hence the unique symmetric equilibrium is

$$q = \pi(\underline{w}) \frac{A(m) - A(0)}{m}$$

and all the surplus from innovations goes to the inventors.

**Case 2: Excess demand of labor** Since  $r > \hat{r}$ , in the labor market equilibrium, a measure  $\hat{r}$  of entrepreneurs hire workers and a measure  $r - \hat{r}$  work alone. For the entrepreneurs working alone their profit from adopting  $h$  innovations is  $\alpha A(k) - kq$ ; hence the previous case implies that they will adopt either all innovations or none. They will adopt if and only if

$$q \leq \alpha \frac{A(m) - A(0)}{m} \tag{10}$$

For any  $u^*$ , the wage chosen by an entrepreneur with technology  $A(k)$  who hires  $n$  workers is  $\max\{\underline{w}, u^*/A(k)\}$ .<sup>24</sup> Let

$$\Pi(k, u^*) = \pi(\max\{\underline{w}, u^*/A(k)\})$$

The value of being an entrepreneur is therefore

$$V(k, u^*) = \max[\alpha, \Pi(k, u^*)]A(k) \tag{11}$$

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<sup>23</sup>Convexity of  $A(k)$  is immediate since the second derivative is  $(\log(1 + \gamma))^2 A(k) > 0$ .

<sup>24</sup>Recall that for a given technology,  $\underline{w}$  is the profit maximizing normalized wage. Hence if  $u^*/A(k) < \underline{w}$ , it is best for the entrepreneur to offer the wage  $\underline{w}$ , that is give a surplus to his workers strictly greater than  $u^*$ .

**Lemma 3.** For any  $u^*$ ,  $V(k, u^*)$  is a strictly increasing and convex function of  $k$ .

*Proof.* Since  $\Pi(k, u^*)$  increases and converges to  $\pi(\underline{w}) > 1$ , there exist cutoff values  $k_0(u^*), k_1(u^*), k_0(u^*) < k_1(u^*)$  such that

$$V(k, u^*) = \begin{cases} \alpha A(k) & \text{if } k \leq k_0(u^*) \\ \pi(u^*/A(k))A(k) & \text{if } k \in [k_0(u^*), k_1(u^*)] \\ \pi(\underline{w})A(k) & \text{if } k \geq k_1(u^*) \end{cases}$$

It is enough to show that  $\pi(u^*/A(k))A(k)$  is convex in order to show that  $V(k, u^*)$  is convex. Differentiating twice, we have

$$\begin{aligned} \frac{d^2\pi(u^*/A(k))A(k)}{dk^2} &= n(\log(1 + \gamma))^2 \left(1 - 4\frac{c\mu}{u^*}A(k)\right) A(k) \\ &\geq n(\log(1 + \gamma))^2 \left(1 - 4\frac{c\mu}{\underline{w}}\right) A(k) \\ &= n(\log(1 + \gamma))^2 (1 - 4\sqrt{c\mu}) A(k) \\ &\geq 0 \end{aligned}$$

where the second inequality is due to  $u^*/A(k) \geq \underline{w}$  and the last inequality by Assumption 2.  $\square$

If condition (10) holds, entrepreneurs who work alone have technology  $A(m)$ ; since they can hire workers if they desire, it must be the case that  $V(m, u^*) = \alpha A(m)$ , that is that  $u^* = \bar{w}A(m)$ .

If condition (10) does not hold, inventors sell only to entrepreneurs who hire workers. By Lemma 3, we can apply the reasoning in the case of excess supply of labor and the unique symmetric equilibrium must satisfy

$$q = \frac{V(m, u^*) - V(0, u^*)}{m}. \quad (12)$$

and entrepreneurs get  $V(0, u^*)$ . Since entrepreneurs who work alone have a profit of  $\alpha A(0)$ , we must have  $V(0, u^*) = \alpha A(0)$  and therefore  $u^* = \bar{w}A(0)$ . Note that entrepreneurs who adopt the  $m$  innovations can hire workers at a wage  $\hat{w} = \bar{w}A(0)/A(m)$ .

At prices  $q$ , entrepreneurs are indifferent between working alone and using tech-

nology  $A(0)$  or hiring workers and using technology  $A(m)$ . The return to an inventor is  $\alpha r q$ . If an inventor sets a price of  $q' < q$ , then  $V(m, u^*) - (m - 1)q - q' > A(0)$  and all entrepreneurs who work alone should purchase the  $m$  inventions and hire workers. This is clearly inconsistent with an equilibrium, therefore it must be the case that condition (10) holds. But then it must hold with an equality by (12).

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