Segregation, Equity and Efficiency

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Abstract: We compare the welfare and equity properties of two compensation rules for university professors: a laissez-faire policy where universities are free to discrimnate betwen professors of different quality and an equity based compensation where wages must be equalized inside a university. In terms of matching *inside* universities, the laissez-faire equilibrium involves heterogeneity of types while the other policy leads to *segregation*. Laissez-faire is always more efficient in terms of surplus and total output. More surpringly the ex-post distribution of wages can be more unequal under the "equity based" compensation than under laissez-faire. This illustrates some possibly unintended consequences of well meaning policy makers' and university administrators' attempts to maintain equity.

1. Introduction

Like most organizations, a university is concerned with its own performance, measured in terms of the quality of its research and teaching, its influence on policy decisions, or the size of its endowment. Even if these goals, which are not always harmonious, do not come into serious conflict, the university's mission is further complicated by the fact that its leaders, unlike most of their counterparts in profit-making firms, typically worry about their "role in society," the performance of the education sector as a whole, and the way the benefits of the sector are distributed among the population of students, faculty, and society at large. As universities shed their "ivory tower" role and become more connected to the rest of the economy, the tensions among their various goals become more apparent.

Of course, interest in these issues is not limited to universities. Economists who study the distribution of income share many of the same concerns and have long recognized the importance of association – who one works with, goes to school with, lives with, or marries – to the determination of individual and aggregate welfare.[1] When the attributes of an organization's members are the most important determination of its performance – surely the case for a university – getting the right association of talented people assumes critical importance. The task is not easy because universities and the individuals who compose them are involved in more or less continual competition with each other to attract talented people into their ranks.

Economists have developed an analytical apparatus – known variously as assignment games or matching models – for studying such situations.[2] These models make an important and robust prediction: when the benefits to joint activity can be divided among the participants in any way without affecting the total (an assumption known as transferable utility, or TU), and the various skills and attributes of the participants are complementary (economist's word for synergistic), then a competitive economy leads to positive assortative matching ("higher" types are matched with "higher" types, lower types with lower ones). Moreover, this outcome is efficient in the sense of maximizing the aggregate benefit over all possible ways of matching people.

The standard argument for positive assortative matching and its efficiency goes like this. Suppose there are two teachers of abilities $a_H > a_L$ and two universities with infrastructures $b_H > b_L$. If a student goes to a university with infrastructure b and with a teacher of ability a then the quality of his final degree is ab, which is for simplicity also equal to his future wage. Teachers and universities have payoffs which are linear in income, and income comes from selling education services to students (who are all identical ex-ante). Complementarity

[1] See for instance [2], [5], [6], [9], [10], [11], [12].

[2] Classical papers are [1], [8], [4].

in production means that the output gain when a_L switches from b_L to b_H is smaller than that gained when a_H makes the same switch. This property is satisfied since $a_H > a_L$ and $b_H > b_L$ imply that

$$a_H b_H - a_H b_L > a_L b_H - a_L b_L,$$

which is the desired complementarity property (the function ab satisfies a condition known as *supermodularity*, which is the most common mathematical representation of the economic idea of complementarity). Therefore, b_H can outbid b_L for the more productive teacher and, in an efficient (or competitive or core) allocation of such an economy, a_H matches with b_H and a_L matches with b_L : we get positive assortative matching of teachers and universities. Note moreover that the previous inequality can also be written

$$a_H b_H + a_L b_L > a_L b_H + a_H b_L,$$

and total output is maximized under positive assortative matching.

A particularly strong form of positive assortative matching is *segregation*, in which members of each "partnership" are identical to each other (of course this concept only makes sense when we are thinking of matches in which both members are the same kind of actor, e.g. two professors, rather than a universities and a teacher). Many people believe that a high degree of segregation is associated with a high level of inequality. Economists studying the recent trends of increasing inequality have therefore focused a lot of attention on whether an increase in segregation might be accompanying the trend and therefore might be causing it.

The rationale behind this argument is that with recent improvements in transportation, communication and especially information technology, impediments to an efficient process of matching should be disappearing: unlike in the past, now little should stop an individual from moving to the university (or firm or spouse) for which he is best suited. Thus if complementarities are empirically significant we should see more segregation at the same time as better aggregate performance. Although this may result in more inequality, according to the theory, that is the price that has to be paid for the best overall performance.

Now it turns out there are different mathematical representations of the economic notion of complementarity, and depending on which one is made, the prediction is slightly different. Segregation is not generally optimal if the joint benefits satisfy a condition known as *weak increasing differences* instead of supermodularity; instead a heterogenous form of positive assortative matching will be optimal and will be the outcome of frictionless matching. We introduce these concepts in a recent paper ([7]).

Moreover, the received story depends on the assumption of *transferability.* The assumption that people can costlessly pay each other off in order to achieve any distribution of mutual benefits is a strong one, to say the least. There are many reasons why this assumption can fail to be satisfied. Foremost among them is *incentives.* For much the same reason that a government cannot tax its citizens arbitrarily (as tax rates increase, the citizens respond by producing less, so government revenue may fall), participants in any sort of joint venture cannot usually vary the distribution of benefits without also affecting how much the participants produce.

But another reason, and the one we shall focus on here, is *distributional policy.* Generally there are two sorts of policies that are used to maintain equity. In the U.S., Title IX specifically requires that men and women receive equal funding for all university activities, notably athletics. Even without government, many top level university administrators, either because of widely accepted codes of practice or perhaps because of "political" pressure from departments within their university try to maintain some level of pay equity. But it is difficult to manage all aspects of faculty remuneration. (Many U.S. and some European universities have given up trying and are now actively encouraging corporate funding and licensing.) There is therefore an external pressure to have "equal" wage compensation or equal treatment *inside* a university, but there can still be unequal compensation across universities.

In other cases governments regulate distribution by setting pay scales that are uniform for all universities. For instance, in Belgium and other countries in Europe, professors are on a salary scale and can forecast quite precisely when and by how much their salary will increase; moreover, the position on the scale is simply a function of their seniority. In this case, there is external pressure to have "equal" wage compensation in the whole university system.

Economic analysis can help understand what likely responses may be in each case. Faced with pay scales, people may reallocate their tie toward other activities (consulting rather than research), or may move to other universities or countries. Similarly, to assess whether Title IX is responsible for the closing of certain men's athletic programs (with the student athletes often moving to other universities), as some people have claimed, one needs an analysis that can systematically keep track of all the forces at work. One can then also assess whether the outcome of the policy is desirable. The model in this paper is fit to capture the forces at play under a Title IX type of constraint but we will comment on the other case (pay scale) at the end of the paper.

In[7], we have extended matching theory to consider what frictionless matches will look like in the presence of non-transferable utility, which is the appropriate context for thinking about the effects of policies like Title IX or of inflexible salary schedules. The results can be surprising to the economist whose intuition has been honed by the TU model and are decidedly relevant to the problems faced by universities.

The questions we shall be addressing are

1. Are policies which try to maintain equity within universities likely to maximize the aggregate output of the university community?

2. Are the distributional goals themselves likely to be achieved?

The answer to both questions turns out to be generally no; the first would not surprise many economists, but the second is less obvious. In both cases, the attempt to regulate the distribution of benefits leads to associational responses by the individuals in the economy that undermine the policy, sometimes so severely as to effect the complete opposite of the policy's goal.

2. A Model

2.1. Equilibrium

The model that we use is quite stylized and is intended to illustrate in a clear way the likely consequences of distributional policies when agents are mobile. Professors are distinguished by their "quality". Quality is unidimensional and is indexed by elements of $[a_L, a_H]$, $a_H > a_L \geq 1$, with a continuous cumulative distribution function F that is strictly increasing on its support.

There are two tasks that are needed to produce education: teaching and research; if a professor of quality a invests in research and a professor of quality b invests in teaching the final quality of the education provided is $a^{\theta}b^{1-\theta}$,where the exponent on the "research" quality can be interpreted as the market relative valuation for research, hence the "profit" that the association between professors a and b can bring a university.[3] Without loss of generality we assume that this profit has to be shared between the two professors. Note that once the two professors are together, who will engage in research and who will engage in teaching is a matter of choice and the maximum profit generated by the association of a and b is

$$\pi(a,b) = \max\left\{a^{\theta}b^{1-\theta}, b^{\theta}a^{1-\theta}\right\}. \tag{1}$$

Intuitively, it is profit maximizing to have the "high" quality professor engage in research (if we had assumed an exponent $\theta < 1/2$ for the research task and an exponent of $1 - \theta > 1/2$ for the teaching

[3] Obviously, in reality professors do some teaching and some research but for simplicity we abstract from this. What matters for the qualitative results is that both education and research are needed and that how professors allocate time between the two tasks influences the final quality of education.

task, the high quality agent should teach; our qualitative predictions are preserved for other values of θ). For expositional convenience we assume $\theta \geq 1/2$.

If there is no restriction on compensation, a university could define wages, one for the "research position", one for the "teaching position". Because the profit is a function of the characteristics of the professors occupying each position, these wages might vary across universities. This makes the computation of equilibrium more difficult than in standard markets since it is necessary to solve simultaneously the assignment problem (which quality of research professor will be matched with a given quality of teaching position) and the price problem (what is the wage that a given quality professor will obtain in the market). For instance, a professor of quality a in a research position at a university with a teaching professor of b might have a different wage than in another university where the teaching professor has quality $c \neq b$. However as can be shown (see [7]), in this class of models, we have *equal treatment* : as long as two agents of quality a occupy the same position (research or teaching), their equilibrium payoff must be the same.

Quite generally, the set of possible wage schedules is constrained first by the total profit that can be distributed[4] and second by whatever structural constraints are imposed. We consider here two cases: first when the university has total liberty in setting its wage schedule, second when it is constrained to treat all professors on an equal basis (but this constraint does not apply across universities). Each case leads to a different *feasible set* that we denote by $V_n(a, b)$ (where n stands for "no constraint") and $V_c(a, b)$ (where c stands for "constraint", like Title IX in the U.S.). Assuming that a professor's utility

[4]This ignores the possibility of government interventions. As long as these interventions are type-independent, i.e., do not depend on the characteristics of the professors, our results continue to hold.

level is equal to his wage we obtain,[5]

$$\begin{aligned} V_n(a,b) &= \{(w(a), w(b)) : w(a) + w(b) \leq \pi(a,b)\} \\ V_c(a,b) &= \left\{(w(a), w(b)) : w(a) = w(b) \leq \frac{\pi(a,b)}{2}\right\}. \end{aligned} \tag{2}$$

Note that V_c is simply the diagonal of V_n : by construction, we will observe a unique wage in a given university if the constraint is imposed, while we might observe different wages if the constraint is not imposed. Hence there is necessarily *more inside equality* with the constraint than without; however, as we shall see, this does not imply that there is *more overall equality* with the constraint.

It is now immediate that as long as a and b are together, it is optimal for them to maximize $\pi(a,b)$ by having a engage in the "research task" if and only if $a \geq b$ (since we assume $\theta = \theta$). An assignment is a map

$$m : [a_L, a_H] \to [a_L, a_H].$$

For technical reasons it is also necessary to impose a measure consistency condition, i.e., that for any set A we have

$$F(m(A)) = F(A). \tag{3}$$

Feasible assignments are those satisfying condition (3).

Definition 1 *An equilibrium consists of a feasible matching function* $m : [a_L, a_H] \to [a_L, a_H]$ *and a wage schedule* $w^* : [a_L, a_H] \to \mathbb{R}$ *satisfying the following conditions.*

(i) $(w^*(a), w^*(m(a))) \in V(a, m(a))$;

(ii) For almost all a *and* b, *there does not exist a feasible wage schedule* $\mathbf{w} \in V(a,b)$ *such that* $w(a) > w^*(a)$ *and* $w(b) > w^*(b)$.

[5] A third case corresponds to the Belgium and other European countries where wages must be equal inside *and across* universities. We do not formally consider this case but will discuss it at the end of the paper.

To an equilibrium corresponds an ex-post distribution of wages where $G(w) = F(\{a : w(a) \leq w\})$. One of the questions that we want to ask here is whether imposing "equality" in wage earnings ex-ante yields more "equality" ex-post than in the absence of the restriction. In other words, if the constraint is imposed in order to generate more equality, will this goal be achieved? A second question concerns the matching response of professors in this case.

We note that *all equilibria are constrained Pareto efficient.* If there were a Pareto improvement, then by definition the coalition consisting of all agents in the economy could find a way to improve upon (or "block") the equilibrium payoff; but since the grand coalition cannot achieve anything more than what two-person coalitions can achieve, a two-person coalition could also block, and this would violate the definition of an equilibrium.

We further note that for each equilibrium coalition $\{a, b\}$, $w(a) + w(b) = \pi(a, b)$ (for otherwise it is possible to increase both types' payoffs).

2.2. Descriptions of Equilibrium

We now provide some definitions useful for characterizing equilibria. Corresponding to the matching function m are the equilibrium coalitions: a coalition $\{a, b\}$ is an equilibrium allocation only if $m(a) = b$.

A simple (and strong) form of monotone matching occurs when each agent matches only with someone like himself, a condition we refer to as *segregation.*

Definition 2 *An equilibrium satisfies segregation if* $m(a) = a$ *for almost every* $a\}$.

Segregation is an extreme kind of equilibrium outcome, since it precludes any heterogeneous matches. A form of matching that allows for heterogeneity but maintains monotonicity, in the sense that the partner of a high type will be higher than the partner of a low type, is positive assortative matching, which we define as follows.

Definition 3 *An equilibrium satisfies positive assortative matching (PAM) if for almost any two equilibrium coalitions* $\{a, b\}$ *and* $\{c, d\}$ *the following is true:*

$$\max(a, b) > \max(c, d) \Longrightarrow \min(a, b) \geq \min(c, d)$$

Another way to state this is to say that for any four types $a > b \geq c > d$, PAM rules out the pairing $(\{a, d\}, \{b, c\})$ from being part of the equilibrium. Note that segregation is a kind of positive assortative matching. In our paper [7] we give the necessary and sufficient conditions for obtaining either type of matching.

2.3. The Case of No Exogenous Constraints

The set V_n in (2) satisfies transferability: the wages can be chosen in any way as long as their sum does not exceed the total profit π. It is convenient to use a modified characteristic function that captures the notion of the potential gains from heterogeneous matching. Define the *segregation payoff* by the maximum equal treatment payoff that a type a can attain by matching with another agent of type a. The terminology refers to the fact that in an economy where all agents have type a equal treatment implies that in equilibrium of that economy all agents obtain their segregation payoff $\frac{\pi(a,a)}{2}$. Here, the segregation payoff is simply

$$u(a) = \frac{\pi(a, a)}{2} = \frac{a}{2}.$$

A concept of surplus is then

$$\begin{aligned} s(a, b) &= \pi(a, b) - (u(a) + u(b)) \\ &= a^{\theta} b^{1-\theta} - \frac{a + b}{2}. \end{aligned}$$

A simple argument shows that a coalition $\{a, b\}$ can form in equilibrium only if $s(a, b) \geq 0$; otherwise one of the types necessarily obtains less than its segregation payoff and would be better off deviating and match with a similar type.

Note for any $a > b \geq c > d$

$$a^{\theta}\left(c^{1-\theta} - d^{1-\theta}\right) > b^{\theta}\left(c^{1-\theta} - d^{1-\theta}\right),$$

or

$$\pi(a,c) - \pi(a,d) > \pi(b,c) - \pi(b,d). \tag{4}$$

This property is called weak increasing differences in [7] and is a weakening of the standard increasing difference condition. This property implies that matches must be positive assortative: indeed, if there are two coalitions $\{a,d\}$ and $\{b,c\}$ with $a > b \geq c > d$ in an equilibrium with payoff w^*, feasibility requires

$$\begin{aligned} w^*(a) + w^*(d) &= \pi(a,d) \\ w^*(b) + w^*(c) &= \pi(b,c) \end{aligned}$$

Now. by (4), we have

$$\sum_{i \in \{a,b,c,d\}} w^*(i) < \pi(a,c) + \pi(b,d)$$

and either $w^*(a) + w^*(c) < \pi(a,c)$ or $w^*(b) + w^*(d) < \pi(b,d)$; in either case there is a coalition that can deviate and "block" the proposed payoff. Note that positive assortative matching is consistent with segregation. However, we can show that types are segregated if and only if $\theta = 1/2$; otherwise types are necessarily matched with different types.

Proposition 4 *The equilibrium match is positive assortative.*

(i) If $\theta = 1/2$ there is segregation;

(ii) If $\theta > 1/2$, no type is segregated.

Proof. (i) If $\theta = 1/2$, $s(a,b) = \sqrt{ab} - \frac{a+b}{2}$ hence, $s(a,b) = -\left(\sqrt{a} - \sqrt{b}\right)^2$. Therefore if $a \neq b$, $s(a,b) < 0$ and for any feasible payoff for coalition $\{a,b\}$ one of the types obtains less than its segregation payoff.

(ii) If $\theta > 1/2$, consider $a \in (a_L, a_H)$ and $\varepsilon > 0$. We have $s(a+\varepsilon, a) = (a+\varepsilon)^\theta a^{1-\theta} - a - \frac{\varepsilon}{2}$. Therefore, $\left.\frac{ds(a+\varepsilon,a)}{d\varepsilon}\right|_{\varepsilon=0} = \theta a^{2(1-\theta)} - \frac{1}{2} > 0$ since $\theta > \frac{1}{2}$ and $a > a_L \geq 1$. Because $s(a,a) = 0$, it follows that there exists $\varepsilon > 0$ such that $s(a+\varepsilon, a) > 0$. The result follows. ■

We noted above that equilibria are always constrained Pareto optimal; here, since there is transferable utility, something much stronger can be asserted, namely that the equilibrium match will maximize the aggregate net output. Observe that if a and b are two types which are not matched to each other in equilibrium, then $w^*(a)+w^*(b) \geq \pi(a,b)$, else the pair (a,b) would "block", i.e., find a profitable deviation. Now, if the equilibrium matching pattern $(a, m(a))$ fails to maximize aggregate net output, there is another measure consistent match $(a, \tilde{m}(a))$ which generates a higher aggregate; for at least some type $\hat{a}$ such that $\tilde{m}(\hat{a}) \neq m(\hat{a})$ we must then have $w^*(\hat{a}) + w^*(\tilde{m}(\hat{a})) < \pi(\hat{a}, \tilde{m}(\hat{a}))$, or the aggregate could not be higher. But then the pair $(\hat{a}, \tilde{m}(\hat{a}))$ would have blocked the original equilibrium. A similar argument can be made for the aggregate surplus, and we have

Proposition 5 *If there is transferable utility, then in equilibrium*
(i) the match maximizes aggregate net output;
(ii) aggregate surplus is also maximized.

3. Exogenous Nontransferability and Redistributive Policy

Consider now a Title IX-style redistributive policy in which the output must be shared equally between the partners (so that the feasible set is what we called $V_c(a,b)$).[6]The frontier for a coalition $\{a,b\}$ is now reduced to the point $(\frac{1}{2}a^\theta b^{1-\theta}, \frac{1}{2}a^\theta b^{1-\theta})$, an extreme form of

[6]The model is partly based on [3].

nontransferability. If $a > b$, $u(a) = \frac{a}{2} > \frac{1}{2}a^{\theta}b^{1-\theta}$; hence, in any heterogenous match the high type obtains less than its segregation payoff, which contradicts the equilibrium condition.

Thus imposing the nontransferability not only reduces the efficiency of the match (by Proposition 5), but also changes the matching pattern. The example also suggests that *forces tending toward more egalitarian sharing will also be forces that tend toward increased segregation.*

One can take this further. Suppose the distribution consists of an atom at 1 and an atom at 2 with respective masses μ and $1-\mu$. With the constraint, the ex-post distribution of wages corresponds to the initial distribution of segregation payoffs, i.e.,.a measure μ of low types obtain $u(1) = 1/2$ and a measure $1-\mu$ of high types obtain $u(2) = 1$.

Consider now the situation without the constraint. Note that $s(2,1) = 2^{\theta} - \frac{3}{2} \geq 0$ if and only if $\theta \geq \frac{\ln 3}{\ln 2} - 1$. Hence, if $\theta > \frac{\ln 3}{\ln 2} - 1$, the equilibrium must involve heterogenous matching (by Proposition 5). Two cases are of interest.

If $\mu < 1/2$, i.e., if there is an oversupply of high types, the low types will have all the surplus in equilibrium and $w(1) = 2^{\theta} - 1$ and $w(2) = 1$. Note that this distribution is more egalitarian than the one under segregation since $w(1) > u(1)$ while $w(2) = u(2)$. Hence the income levels are more compressed under laissez faire: *when the low types are scarce, and the policy of equal sharing within coalitions is imposed, not only is there more segregation, but there is also more inequality.*

Of course, if the 1's are more plentiful, then the reverse is true: the equal sharing rule leads to segregation, but also leads to a more egalitarian distribution ($w(1) = 1/2$ and $w(2) = 2^{\theta} - 1/2$ under laissez faire vs. 1/2 and 1 under equal share). This underscores a different point: *greater segregation does not necessarily lead to greater inequality.*

3.1. Other Sources of Nontransferability

Non transferability need not result from sharing rules imposed from outside. It can originate in features of the technology: perhaps the partnership produces a “local public good” like a lecture or seminar – once inside a lecture hall, everyone in it shares its attributes more or less equally. Or it can arise as a result of difficulties in the provision of incentives to the organization’s members to work toward its goals. The analysis of the first case is exactly the same as that we present here, but the policy implication is almost the opposite: a more efficient matching outcome, as well perhaps as a more equitable distribution of the benefits, may be achieved by forgoing the laissez faire match and instead imposing one from outside (economists are fond of pointing out the practical difficulties of getting this right – there is a very real possibility of doing more harm than good this way).

One complication of the second case is the introduction of monitoring costs – resources devoted to gathering information about how individuals perform but which have no direct social benefit. This seems to be a major concern of universities and a bane of faculty who seem to spend more time accounting for how they spend their time on research than doing the research itself! Though the analysis of this situation is more subtle, the general conclusion is the same: the laissez faire outcome will be inefficient as well as inequitable, and there are policies of imposed matching and sharing of output that can both enhance aggregate performance and reduce inequality. Moreover, some of the efficiency gains are attributable to reductions in monitoring costs. We consider this case in detail as the “Incentive Problems” example in **??**.

Finally, when the equal sharing condition is imposed across universities, like in Belgium, there is little room for matching of the sort that we have considered here. In particular, since the wage is independent of the match, professors will tend to locate in universities following geographical, family, or other factors that are not related to “quality.” Total surplus will clearly decrease with respect to either of the cases that we considered previously. Inside the current model, equality will be maximum[7]. However we can go beyond the current model and consider the possibility for professors to engage into non-university related activities, at the expense of their research or teaching duties. It is then simple to show that not only aggregate performance of the university will decrease even more but that inequality might be increased if the "value" of these outside activities is a non trivial function of quality.

4. Conclusion

In this note we have considered some of the forces at work in matching and the distribution of income, focusing particularly on the effect of imposing equitable sharing within organizations. While abstract and very much simplified, this analysis illustrates some possibly unintended consequences of well meaning policy makers' and university administrators' attempts to maintain equity. At the same time, it warns against a too-easy interpretation of perceived trends in matching patterns (e.g. tendencies toward greater segregation) as a reflection of market forces pushing us toward greater efficiency.

Our specific conclusions from the above analysis are:

- even if there are strong complementarities, greater segregation need not imply greater efficiency in the sense of larger aggregate benefits
- there may in some circumstances be justification for some form of directed integration
- greater segregation is not necessarily associated with greater inequality
- imposing pay equity within universities will tend to lead toward greater segregation and often will reduce aggregate performance

[7] Note that absent state subsidies or cross-subsidization the wage must be equal to $\pi(a_L, a_L)/2$.

- segregation therefore need not be the consequence of "better matching" but may instead be the outcome of inefficiencies in the generation and sharing of benefits that lead to *less* efficient matching

- in cases where greater segregation *does* lead to greater inequality, we conclude paradoxically that greater pay equity within universities leads not only to lower aggregate welfare but also to greater overall inequality

The fact that non transferabilities is a pervasive fact of life not only makes analysis more subtle and complex, but makes effective policy more difficult. In particular, if one wants to balance competitiveness and equity considerations in a university system, it is important to realize that imposing equity "by fiat" might fail to achieve this goal in a world where agents are mobile.

5. Reference

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